

This is a digital copy of a book that was preserved for generations on library shelves before it was carefully scanned by Google as part of a project to make the world's books discoverable online.

It has survived long enough for the copyright to expire and the book to enter the public domain. A public domain book is one that was never subject to copyright or whose legal copyright term has expired. Whether a book is in the public domain may vary country to country. Public domain books are our gateways to the past, representing a wealth of history, culture and knowledge that's often difficult to discover.

Marks, notations and other marginalia present in the original volume will appear in this file - a reminder of this book's long journey from the publisher to a library and finally to you.

Usage guidelines

Google is proud to partner with libraries to digitize public domain materials and make them widely accessible. Public domain books belong to the public and we are merely their custodians. Nevertheless, this work is expensive, so in order to keep providing this resource, we have taken steps to prevent abuse by commercial parties, including placing technical restrictions on automated querying.

We also ask that you:

+ *Make non-commercial use of the files* We designed Google Book Search for use by individuals, and we request that you use these files for personal, non-commercial purposes.

+ *Refrain from automated querying* Do not send automated queries of any sort to Google's system: If you are conducting research on machine translation, optical character recognition or other areas where access to a large amount of text is helpful, please contact us. We encourage the use of public domain materials for these purposes and may be able to help.

+ *Maintain attribution* The Google "watermark" you see on each file is essential for informing people about this project and helping them find additional materials through Google Book Search. Please do not remove it.

+ *Keep it legal* Whatever your use, remember that you are responsible for ensuring that what you are doing is legal. Do not assume that just because we believe a book is in the public domain for users in the United States, that the work is also in the public domain for users in other countries. Whether a book is still in copyright varies from country to country, and we can't offer guidance on whether any specific use of any specific book is allowed. Please do not assume that a book's appearance in Google Book Search means it can be used in any manner anywhere in the world. Copyright infringement liability can be quite severe.

About Google Book Search

Google's mission is to organize the world's information and to make it universally accessible and useful. Google Book Search helps readers discover the world's books while helping authors and publishers reach new audiences. You can search through the full text of this book on the web at `http://books.google.com/`

NICHOLS NEW GRADED LESSONS IN ARITHMETIC

PART THREE

Educ T 119.11.605 (3)



Harvard College Library
THE GIFT OF
GINN AND COMPANY


3 2044 097 007 215

NEW GRADED LESSONS

IN

ARITHMETIC

PART III

BY

WILBUR F. NICHOLS, A.M.

SUPERVISING PRINCIPAL OF THE EATON SCHOOL DISTRICT,
NEW HAVEN, CONN.

THOMPSON BROWN COMPANY

(JOHNSON, BLAGDEN & CO.)

BOSTON CHICAGO NEW YORK

Educ T 119.11.605

HARVARD COLLEGE LI
GIFT OF
GINN & COMP
MARCH 17,

Copyright, 1911
By Wilbur F. Nichols

PREFACE

THIS book is intended for the use of pupils in the seventh and eighth years of school.

In the first section of the book, after reviewing the important topics presented in the earlier books of the series, especial emphasis is given to percentage and its applications, including bank discount, commercial discount, insurance, commission, taxes, etc. Great care has been taken to present these topics in a clear and simple manner.

Several lessons are given that bear upon the work in manual training, thus connecting that important branch with the regular work of the schoolroom. For the same reason a few lessons are given in home furnishing and in domestic economy.

The second section is intended for use in the last year of the grammar school, but may be used in those high schools where arithmetic is reviewed. For this reason it does not follow the plan of the preceding sections of the series. With the exception of the topics fully presented in the first part it is a brief but complete arithmetic presented topically.

The aim is to give a final review of all subjects, that the pupils may have an opportunity to clear up any difficult points and gain a complete mastery of the whole subject.

It retains the frequent reviews, which have been found so helpful to pupils in testing their proficiency and developing their power of reasoning.

Few teachers will find any need for supplementary work, as so large a number of problems is given. On the other hand, few pupils should be required to solve all the problems. It is a good plan to assign to all the class, as required work, that number of examples which even the slowest child can do, and then allow any pupil to solve the remaining examples of the lesson as optional work.

In preparing his manuscript the author acknowledges his indebtedness to many teachers for suggestions, but especially to Lincoln W. Barnes, instructor in manual training, New Haven, for the lessons connected with his department; to Robert W. Elliott, Baltimore, Md.; to Miss Sara Knouse, departmental teacher of mathematics, New Haven; and to Charles G. Wetherbee, Prince School, Boston.

WILBUR F. NICHOLS.

TABLE OF CONTENTS

SECTION 1

GENERAL REVIEW

ADDITION PROBLEMS . . . . . Lesson 2
AREA OF CIRCLES . . . . . Lesson 14
AREA OF QUADRILATERALS . . . . . Lesson 13
BOARD MEASURE . . . . . Lesson 15
CARPETING . . . . . Lesson 16
CIRCLES . . . . . Lesson 14
DECIMAL FRACTIONS . . . . . Lessons 6, 7
FRACTIONS . . . . . Lessons 4, 5
MEASUREMENTS . . . . . Lessons 8 to 17
MEASUREMENT OF ANGLES . . . . . Lesson 10
MEASUREMENT OF CARPETING . . . . . Lesson 16
MEASUREMENT OF CIRCLES . . . . . Lesson 14
MEASUREMENT OF QUADRILATERALS . . . . . Lesson 13
MEASUREMENT OF TRIANGLES . . . . . Lesson 11
NOTATION . . . . . Lesson 1
NUMERATION . . . . . Lesson 1
ORAL . . . . . Lessons 4, 6, 8
PAPERING . . . . . Lesson 17
PLASTERING . . . . . Lesson 9
PROBLEMS . . . . . Lessons 2, 3
QUADRILATERALS . . . . . Lessons 12, 13
SUBTRACTION PROBLEMS . . . . . Lesson 5

ADVANCED WORK

ALGEBRAIC EQUATIONS . . . . . Lessons 46 to 53
ANALYSIS . . . . . Lessons 55, 56
ANGLES . . . . . Lesson 94
BANK DISCOUNT . . . . . Lessons 98 to 103
BINS, CAPACITY OF . . . . . Lesson 18

CHECKS . . . . . Lesson 99
COMMERCIAL DISCOUNT . . . . . Lessons 34, 35, 36
COMMISSION . . . . . Lessons 31, 32, 33
COMPOUND INTEREST . . . . . Lessons 82 to 85
CONSTRUCTION . . . . . Lessons 93, 95
CYLINDERS . . . . . Lesson 19
DOMESTIC SCIENCE . . . . . Lessons 64, 65
ENGLISH MONEY . . . . . Lesson 67
EQUATION . . . . . Lessons 46 to 53
ESTIMATING DISTANCES . . . . . Lesson 66
FRANC . . . . . Lesson 68
HOME FURNISHING . . . . . Lesson 65
INSURANCE . . . . . Lessons 28, 29, 30
INTEREST . . . . . Lessons 38 to 41, 105
LIRA . . . . . Lesson 68
LONGITUDE AND TIME . . . . . Lessons 72 to 78
MANUAL TRAINING . . . . . Lessons 60 to 63
MARK . . . . . Lesson 68
MEASUREMENTS . . . . . Lessons 20, 66, 70, 71, 90
MEASURES OF VALUE . . . . . Lessons 67, 68
MISCELLANEOUS PROBLEMS . . . Lessons 21, 23, 45, 69, 79, 97, 107, 109, 111, 112, 114
NOTES . . . . . Lesson 104
ORAL . . . Lessons 22, 28, 30, 42, 44, 46, 50, 52, 54, 55, 58, 70, 80, 88, 90, 96, 106, 108, 110, 113
PAPERING . . . . . Lesson 17
PARTITIVE PROPORTION . . . . . Lesson 57
PERCENTAGE . . . . . Lessons 24 to 37
PFENNIG . . . . . Lesson 68
PLASTERING . . . . . Lesson 9
PROFIT AND LOSS . . . . . Lessons 26, 27
PROPORTION . . . . . Lessons 55, 56, 57
RATIO . . . . . Lesson 54
RECEIPTS . . . . . Lesson 104
SOVEREIGN . . . . . Lesson 67
STANDARD TIME . . . . . Lessons 77, 78
STATEMENTS . . . . . Lessons 43, 59, 81
STUDY OF LINES . . . . . Lessons 91, 92
SURFACE OF CYLINDERS . . . . . Lesson 19
TAXES . . . . . Lessons 86 to 89
VOLUME OF CYLINDERS . . . . . Lesson 19

SECTION 2

ACCOUNTS . . . . . Lessons 140, 142, 143
ADDITION . . . . . Lessons 120, 121
ADDITION OF COMPOUNDS NUMBER . . . . . Lesson 173
ADDITION OF FRACTIONS . . . . . Lessons 152, 153
ALGEBRA . . . . . Lessons 212 to 219, 254, 255
AREA OF PARALLELOGRAMS . . . . . Lesson 177
BILLS . . . . . Lessons 140, 141, 143
BONDS . . . . . Lessons 203 to 207
BRICK WORK . . . . . Lesson 184
BUSINESS FRACTIONS . . . . . Lessons 166, 167
CANCELLATION . . . . . Lesson 147
CASH ACCOUNTS . . . . . Lesson 142
COMPOUND NUMBERS . . . . . Lessons 170 to 175
CONES . . . . . Lessons 182, 183
CONSTRUCTION . . . . . Lessons 190, 191
COST BY C. M. AND T. . . . . Lesson 169
CUBE ROOT . . . . . Lessons 251, 252, 253
CUSTOMS . . . . . Lessons 194, 195
CYLINDERS . . . . . Lessons 180, 181
DECIMALS . . . . . Lessons 130 to 135
DEFINITIONS . . . . . Lessons 115, 148, 170, 176
DENOMINATE NUMBERS . . . . . Lessons 170 to 175
DIVISION . . . . . Lessons 127, 128, 129
DIVISION OF COMPOUND NUMBERS . . . . . Lesson 175
DIVISION OF DECIMALS . . . . . Lessons 132, 133
DIVISION OF FRACTIONS . . . . . Lessons 158, 159, 160
DUTIES . . . . . Lessons 194, 195
EQUATIONS . . . . . Lessons 212 to 219
EXACT INTEREST . . . . . Lesson 250
EXCHANGE . . . . . Lessons 228 to 232
FACTORING . . . . . Lesson 144
FACTORS . . . . . Lessons 144, 145
FARM PROBLEMS . . . . . Lesson 241
FRACTIONS, COMMON . . . . . Lessons 148 to 163, 166, 167
FRACTIONS, DECIMAL . . . . . Lessons 130 to 135
GEOMETRY . . . . . Lessons 190, 191
GREATEST COMMON DIVISOR . . . . . Lesson 145
INTEREST . . . . . Lessons 202, 250
LATHING . . . . . Lesson 178
LEAST COMMON MULTIPLE . . . . . Lesson 146

MEASUREMENTS . . . . . Lessons 176 to 189
METRIC SYSTEM . . . . . Lessons 233 to 237
MISCELLANEOUS PROBLEMS . . Lessons 137, 165, 193, 197, 199, 209, 239, 240, 243, 245
MULTIPLICATION . . . . . Lessons 124, 125, 126
MULTIPLICATION OF COMPOUND NUMBERS . . . . . Lesson 174
MULTIPLICATION OF DECIMALS . . . . . Lesson 132
MULTIPLICATION OF FRACTIONS . . . . . Lessons 154 to 157
NOTATION AND NUMERATION . . . . . Lessons 116 to 119

ORAL

ALGEBRA . . . . . Lesson 214
DENOMINATE NUMBERS . . . . . Lesson 171
FRACTIONS . . . . . Lessons 162, 168
MEASUREMENTS . . . . . Lessons 179, 186, 188
MISCELLANEOUS . . Lessons 136, 164, 192, 196, 198, 208, 238, 242, 244
PERCENTAGE . . . . . Lesson 200
PARALLELOGRAMS . . . . . Lesson 177
PARTIAL PAYMENTS . . . . . Lessons 248, 249
PARTNERSHIP . . . . . Lessons 210, 211
PAVING . . . . . Lesson 178
PERCENTAGE . . . . . Lesson 201
POWERS AND ROOTS . . . . . Lessons 220, 221
PRISMS . . . . . Lesson 180

PROBLEMS IN

DECIMALS . . . . . Lessons 134, 135
DIVISION . . . . . Lesson 129
FRACTIONS . . . . . Lessons 157, 161, 163
INTEREST . . . . . Lessons 246, 247
MEASUREMENTS . . . . . Lessons 187, 189
MISCELLANEOUS . . . . Lessons 137, 165, 193, 197, 199, 209, 239, 240, 243, 245
MULTIPLICATION . . . . . Lesson 126
PERCENTAGE . . . . . Lesson 201
UNITED STATES MONEY . . . . . Lesson 139
PYRAMIDS . . . . . Lesson 182
REDUCTION OF DENOMINATE NUMBERS . . . . Lessons 170, 171, 172
REDUCTION OF FRACTIONS . . . . . Lessons 149 to 152
REVIEW . . . . Lessons 137, 165, 193, 197, 199, 209, 239, 240, 243, 245
ROMAN NOTATION . . . . . Lesson 119

ROOFING . . . . . Lesson 178
ROOTS AND POWERS . . . . . Lessons 220, 221
STOCKS AND BONDS . . . . . Lessons 203 to 207
STONE WORK . . . . . Lesson 184
SQUARE ROOT . . . . . Lessons 220 to 227
SUBTRACTION . . . . . Lessons 122, 123
SUBTRACTION OF COMPOUND NUMBERS . . . . . Lesson 173
SUBTRACTION OF FRACTIONS . . . . . Lesson 153

SURFACE OF
CONES . . . . . Lessons 182, 183
PYRAMIDS . . . . . Lessons 182, 183
SPHERES . . . . . Lesson 185
TABLES OF WEIGHTS AND MEASURES . . . . . Lessons 256 to 258
UNITED STATES MONEY . . . . . Lessons 138, 139

VOLUME OF
CYLINDERS . . . . . Lessons 180, 181
PRISMS . . . . . Lesson 180
SPHERES . . . . . Lesson 185

SECTION I

LESSON 1. *NOTATION AND NUMERATION. REVIEW*

You are supposed to be familiar with the reading and writing of numbers.

1. Name the ten figures, called digits, used in the Arabic system of notation.

2. Why do we call this system a *decimal* system?

3. What is numeration?

4. On what does the value of any figure depend?

5. What change is made in the value of a figure if it is moved one place to the left? One place to the right?

6. In the number 4236, for what does the 6 stand? For what does the 3 stand? The 2? The 4?

7. Name the first four periods in order.

8. You will seldom use a larger number, yet it may be well to learn the names of the higher periods, viz.: trillions, quadrillions, quintillions, sextillions, septillions, octillions, nonillions, decillions.

9. In a number like 26.3, what do we call the period (.)?

Read the following numbers:

10. 1,401.00405
11. 2,501,072,010.0004
12. 760.010402
13. 36,105,140,715.00412
14. 78.000107
15. 11,101,000,010.000702

16. Notice the similarity of sound when reading the following:

101,000, one hundred one thousand.
.101, one hundred one thousandths.
100.001, one hundred *and* one thousandth.

In 1908 the appropriations for the different departments in a certain city were as follows:

Education	$600,181.00	Public Works	$253,794.00
Police	224,140.00	Fire	199,525.00
Charities	92,690.00	Lighting	80,923.00
Parks	28,000.00	Health	26,350.00
Library	19,500.00	Assessors	11,580.00
Collector	8,780.00	Town Clerk	9,080.00
Corporation Counsel	6,300.00	Registrar	5,990.00
Board of Relief	900.00	Interest	166,010.26
Sinking Fund	100,000.00	Miscellaneous	190,437.35

1. Find the total amount of the appropriations.

2. How much more was allowed for education than for police and fire protection?

3. How much more was appropriated for public works than for the interest on the debt?

4. How much less was appropriated for lighting than for charities?

5. How much more did the park commissioners receive than the board of health?

AREAS AND POPULATION OF STATES

States	Area in square miles	Population, 1900
Maine	33,040	694,466
New Hampshire	9,305	411,588
Vermont	9,565	343,641
Massachusetts	8,315	2,805,346
Rhode Island	1,250	428,556
Connecticut	4,990	908,420
New York	49,170	7,268,894
New Jersey	7,815	1,883,699
Maryland	12,219	1,188,044

6. Find the total area and total population of these states.

7. How much larger is the population of New York than the total population of the New England States?

8. Make 10 examples for the class to solve based on these statistics.

1. A grocer paid \$1,148.40 for flour at \$4.35 a barrel. If he sold it at \$5.15 a barrel, how much did he gain?

2. The area of Porto Rico is 3606 sq. mi. and the population is 953,243. Find to the nearest unit the population of each square mile.

3. Porto Rico exports annually \$14,867,000 worth of goods. Find to the nearest dollar the amount of exports for each person.

4. The area of France is 207,054 sq. mi. and the value of her exports is \$1,119,400,000. Find to the nearest dollar the value of the exports for each square mile.

A realty company issues the following statements:

Assets		Liabilities	
Real Estate	\$784,187.25	Bond Accounts	\$233,896.63
Office Equipment	5,083.46	Profit Sharing Bonds	62,335.00
Loans	5,923.65	Bonds	5,865.00
Sundry Accounts	14,583.11	Real Estate	213,850.00
Cash	14,144.57	Sundry Accounts	2,212.07
		Capital Stock	100,000.00

5. Find the value of the assets. Find the value of the liabilities.

6. Subtract the liabilities from the assets to find the profits.

7. Two automobiles start from two towns 105 miles apart and travel toward each other. They meet in 3 hours. If the first traveled 15 miles an hour, how many miles did the second travel an hour?

8. A dealer sold 96 yd. of carpeting at \$1.87$\frac{1}{2}$ a yard. If he gained \$38.40, how much did it cost him a yard?

9. How many farms containing 90 acres each can be formed in a town that is in the form of a rectangle 6 miles long and 4$\frac{1}{2}$ miles wide? There are 640 acres in a square mile.

10. If 63 is added to a certain number, it will contain forty-two 246 times. What is the number?

You have studied fractions so long that you should be familiar with all the definitions and processes. Therefore explanations are omitted from this book.

1. What is a fraction?

2. Tell the meaning of $\frac{2}{3}$, $\frac{3}{5}$, $\frac{5}{7}$, $\frac{7}{9}$.

3. Which term of the fraction gives the size of the parts and therefore names the fraction?

4. Which term of the fraction numbers the parts?

a. $\frac{3}{4}$	*b.* $\frac{7}{8}$	*c.* $2\frac{1}{2}$	*d.* $\frac{7}{6}$	*e.* $4\frac{1}{3}$	*f.* $\frac{5}{6}$
g. $\frac{12}{4}$	*h.* $12\frac{1}{5}$	*i.* $\frac{1}{2}$	*j.* $7\frac{1}{4}$	*k.* $\frac{11}{5}$	*l.* $7\frac{1}{6}$

5. Of these fractions name the improper fractions. What is an improper fraction?

6. Apply the definition of a fraction to $\frac{4}{3}$, and state why it might be called an incorrect fraction.

7. What is a mixed number? In the list above name the mixed numbers.

8. What do you mean by reduction of fractions?

9. Change $\frac{6}{12}$ to halves. What principle is illustrated when fractions are reduced to their lowest terms?

10. Change $\frac{2}{3}$ to ninths. What principle of fractions is illustrated when fractions are changed to higher terms?

11. What method did you learn in Lesson 9 for reducing a fraction to its lowest terms?

Reduce to lowest terms:

12. $\frac{8}{12}$	**13.** $\frac{18}{24}$	**14.** $\frac{21}{35}$	**15.** $\frac{14}{28}$	**16.** $\frac{15}{25}$

Reduce to 24ths :

17. $\frac{2}{3}$	**18.** $\frac{1}{2}$	**19.** $\frac{3}{4}$	**20.** $\frac{5}{6}$	**21.** $\frac{7}{8}$

22. How do you change a mixed number to an improper fraction?

Change to improper fractions :

23. $3\frac{1}{2}$	**24.** $4\frac{1}{4}$	**25.** $5\frac{1}{5}$	**26.** $6\frac{1}{6}$	**27.** $7\frac{1}{7}$

1. What are similar fractions?

2. Add, *a.* $4\frac{5}{7}$, $7\frac{2}{3}$, $5\frac{4}{5}$. *b.* $1\frac{7}{8}$, $2\frac{3}{4}$, $4\frac{7}{12}$, $5\frac{1}{6}$.

3. Explain addition of fractions, and give the reasons for every step.

4. The three sides of a triangle are $5\frac{2}{3}$ inches, $3\frac{1}{2}$ inches and $6\frac{3}{16}$ inches. What is the perimeter?

5. From $56\frac{1}{3}$ take $17\frac{7}{12}$.

6. What number must be subtracted from $126\frac{1}{4}$ to leave $37\frac{7}{8}$?

7. A man spent $\frac{1}{4}$ of his income for rent, $\frac{3}{8}$ for food, and $\frac{3}{16}$ for clothing. What part of his income did he spend? What part of his income was left?

8. How do you multiply an integer by a fraction?

9. At \$16 a ton, how much will $9\frac{3}{4}$ tons of hay cost?

10. State the method of multiplying a fraction by an integer.

11. Find the perimeter of a square that measures $15\frac{7}{12}$ ft.

12. What is cancelation?

13. How do you multiply a fraction by a fraction?

Using cancelation find the product of:

14. $\frac{7}{8} \times \frac{3}{3}\frac{2}{5}$ 15. $\frac{5}{16} \times \frac{4}{5}\frac{8}{5}$ 16. $\frac{1}{1}\frac{1}{2} \times \frac{3}{5}$

17. $27\frac{1}{3} \times 33\frac{2}{3}$ 18. $6\frac{1}{2} \times 24\frac{1}{3}$ 19. $12\frac{2}{9} \times 27\frac{1}{8}$

20. Find the cost of $15\frac{1}{2}$ tons of hay at $\$18\frac{3}{4}$ a ton.

21. If a bird can fly $9\frac{1}{4}$ miles in $\frac{1}{3}$ of an hour, how far can it fly in $1\frac{1}{2}$ hours?

22. Find $\frac{1}{4}\frac{9}{2}$ of 5208. Find $\frac{2}{3}\frac{6}{7}$ of 444.

23. A garden is $15\frac{2}{3}$ yd. long and $9\frac{5}{6}$ yd. wide. How many square yards are there in the area of the garden?

24. A strong wind giving a pressure of $14\frac{1}{2}$ lb. per square foot blows squarely against the end of a building 24 ft. wide and $18\frac{1}{2}$ ft. high. What is the total wind pressure against the end of the building?

1. Read, 300.03; 5,000.005; 7.2.

2. Read the following as *a*, mixed decimals. *b*, as thousandths: 2.007; 26.027; 250.305.

3. Name the decimal places in order from tenths to billionths.

4. How do you multiply a decimal by 10? By 100?

5. How do you divide a decimal by 10? By 1000?

6. Prefixing a cipher to a decimal has what effect upon the value of the decimal? Why?

7. How should decimals be arranged for convenience in adding or subtracting?

8. Add 2.2; 1.01; 3.21.

9. From 4.4 take 3.21.

10. How do the number of decimal places in the product compare with the number in both multiplicand and multiplier?

Multiply:

11. 25 by .1	12. 2.5 by .1	13. .25 by .1
14. 15 by .01	15. 1.5 by .01	16. .15 by .01

17. In dividing a decimal by a decimal what change should you first make in the position of the decimal points?

18. Can you tell why changing the position of the points as you have indicated does not change the value of the quotient?

19. Divide 2.5 by .5. After changing the points how does this example read?

Divide :

20. .12 by .4	22. 1.2 by .4	24. .012 by .4
21. 2.4 by .6	23. .24 by .06	25. .024 by .06

26. A unit consists of how many hundredths? State this question in another form.

27. How many thousandths make a tenth?

1. Divide 9.936 by .46.

$$
\begin{array}{r}
21.6 \\
46\overline{)993.6} \\
92 \\
\hline
73 \\
46 \\
\hline
276 \\
276 \\
\hline
\end{array}
$$

Before dividing change the divisor to an integer by moving the decimal points in both dividend and divisor as many places as there are decimal figures in the divisor. Place the decimal point in the quotient directly above the point in the dividend.

Find the *specific gravity* of the following substances by dividing the weight of a cubic foot as given by 62.5 lb., the weight of a cubic foot of water. Carry answer to thousandths.

2. Glass, 175.8 lb.
3. Maple Wood, 46.886 lb.
4. Silver, 655.1 lb.
5. Brick, 118.3 lb.
6. Brass, 523.8 lb.
7. Cork, 15 lb.

8. What will 16.375 casks of wine cost, if 8.1875 casks cost $982.50?

9. Divide 234.61875 by $16\frac{1}{8}$.

10. If .75 of a yard of cloth cost $2.40, how many yards can be bought for $38.40?

11. The sum of two numbers is $127\frac{7}{8}$, and the smaller number is 49.125. Find the greater number.

12. If a man can build .425 of a rod of fence in an hour, how many rods can 12 men build in 6.5 days, working 8.25 hours a day?

13. If .35 of a cargo of 8,000 bu. of wheat were burned, what was the value of the part left at $.875 a bushel?

14. Add: twenty-six and fifteen thousandths; eighty-one thousand nine hundred ten-thousandths; eleven and twenty thousand seven hundred four hundred-millionths; twelve hundred and twelve hundred-thousandths.

1. How do you find the area of a rectangle?

2. How many square inches are there on the surface of a 6-in. cube?

3. How many 4-in. squares can be cut from a 20-in. square?

4. If the area of a rectangular field is 77 sq. rd., and the base is 11 rods, what is the altitude?

5. How many square rods are there in a field 80 rd. long and 40 rd. wide? How many acres are there?

6. How many cubic feet are there in a cord of wood?

7. What are the dimensions of a cord of wood?

8. A pile of wood is 32 ft. long, 4 ft. high, and 4 ft. wide. Find the number of cords in the pile without finding the number of cubic feet.

9. A block of granite is 4 ft. long, 3 ft. wide, and 2.5 ft. thick. How many cubic feet does it contain?

10. At \$1.25 a cubic foot find the cost of the granite block in the eleventh example.

11. How many square yards are there in a ceiling 18 ft. by 24 ft? At 20¢ a square yard, find the cost of painting the ceiling.

12. How many cubic feet are there in a rectangular block 2 ft. square at the end and 6 ft. long?

13. How many times larger would a block be that was twice as long, twice as wide, and twice as thick?

14. How many blocks $\frac{1}{4}$ of an inch on a side can be sawed from a 2-in. cube?

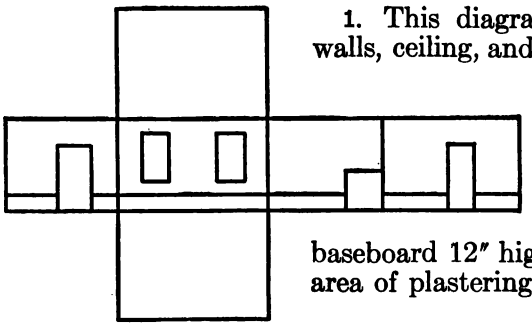
15. How many square rods are there in a rectangular garden, 8 rd. long and 5 rd. wide?

16. What is this garden worth at \$2$\frac{1}{2}$ a square rod?

17. What will it cost to fence this garden at \$1.25 a rod?

18. Give the area of a rectangle measuring 7$\frac{1}{2}$ inches by 6 inches.

In estimating the cost of painting or plastering in most places it is the custom to deduct from the total area one half of the area of the openings. Since this rule varies, you should be guided by the custom observed in your locality.



1. This diagram represents the walls, ceiling, and floor; 16′ long 12′ wide, 9′ high. There are 2 doors, each 4′ by 7′; 2 windows, each 4′ by 6′; a fireplace 5′ by 5′, and a baseboard 12″ high. Find the total area of plastering.

$2 \times (16' + 12') = 56'$ perimeter.
56 sq. ft. $\times$ 9 = 504 sq. ft. in walls.
16 sq. ft. $\times$ 12 = 192 sq. ft. in ceiling.
504 sq. ft. + 192 sq. ft. = 696 sq. ft. total area.
$2 \times (7$ sq. ft. $\times 4) = 56$ sq. ft. in doors.
$2 \times (6$ sq. ft. $\times 4) = 48$ sq. ft. in windows.
5 sq. ft. $\times$ 5 = 25 sq. ft. in fireplace.
56 ft. $- (4' + 4' + 5') = 43'$ length of baseboard.
43 sq. ft. $\times$ 1 = 43 sq. ft. in baseboard.
$\frac{1}{2}(56 + 48 + 25) + 43$ sq. ft. $= 107\frac{1}{2}$ sq. ft. to be deducted. 696 sq. ft. $- 107\frac{1}{2}$ sq. ft. $= 588\frac{1}{2}$ sq. ft. of plastering.

2. At 20¢ a square yard find the cost of plastering a room 20′ long, 17′ wide, and 12′ high. There are 4 doors each 7′ by $4\frac{1}{2}'$, 3 windows each 6′ by $4\frac{1}{2}'$, and a baseboard 8 in. high.

3. A room is 18′ by 24′ by 9′. There are 2 doors 4′ by $7\frac{1}{2}'$, 2 windows, 4′ by 6′, and a baseboard 9 in. high. Find the cost at 22¢ a square yard of plastering the room.

4. Find the cost at 2¢ a square yard of kalsomining the walls and ceiling of a room 16′ by 15′ by 10′, when no allowance is made for openings.

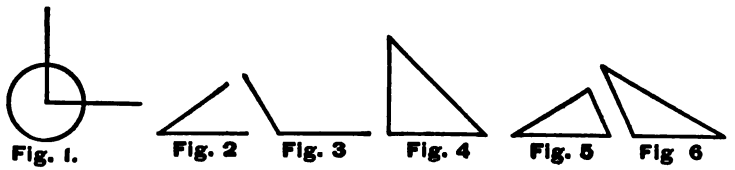


Fig. 1. Fig. 2 Fig. 3 Fig. 4 Fig. 5 Fig 6

1. How many degrees are there in a circle?

2. What part of the whole circle does it take to measure the angle in Fig. 1?

3. Then Fig. 1 is an angle of how many degrees?

4. It is also called a **right angle.**

5. An **angle** is the difference in direction of two straight lines that meet or would meet if produced.

6. An **acute angle** is an angle smaller than a right angle. Which figure is an acute angle?

7. An **obtuse angle** is an angle greater than a right angle. Which figure is an obtuse angle?

8. A **triangle** is a plane figure bounded by three straight lines.

9. Triangles are named from their largest angles, as **acute triangle, right triangle,** and **obtuse triangle.** Name the triangles at the top of the page.

10. Angles are measured by means of a protractor, which is half a circle, whose circumference is marked off into degrees.

11. Cut from cardboard a protractor and use it in measuring all the angles on this page.

TABLE OF ANGLES AND CIRCLES

60 seconds (″) = 1 minute (′)
60 minutes = 1 degree (°)
360 degrees = 1 circumference

12. Any part of the circumference of a circle is an **arc.** One quarter of the circumference is a **quadrant.**

1. Triangles are also named from their sides as, equilateral, isosceles, and scalene.

2. An **equilateral** triangle has 3 equal sides.
An **isosceles** triangle has 2 equal sides.
A **scalene** triangle has none of its sides equal.

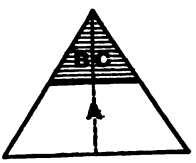


Fig. 1

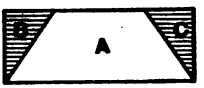


Fig. 2

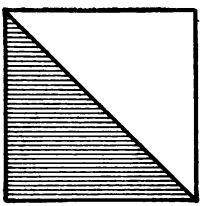


Fig. 3

3. The **base** of a triangle is the side upon which it rests.

4. The **vertex** is the angle opposite the base.

5. The **altitude** is the perpendicular distance from the base, or the base produced, to the vertex.

6. From Fig. 3, can you see that the area of a triangle is equal to $\frac{1}{2}$ the area of a rectangle having the same dimensions?

7. From Fig. 1 and 2, can you see that the area of a triangle is equal to the area of a rectangle having the same base and $\frac{1}{2}$ of the altitude?

Learn: The area of a triangle is equal to half the area of a rectangle having the same base and altitude.

Find the area of triangles with the following bases and altitudes:

8. 36 ft., 24 ft.
9. 2′ 6″, 3′.
10. 17 yd., 15 yd.
11. 3 yd. 1 ft., 4 ft.
12. 19 rd., 13 rd.
13. 7 ft. 8 in., 5 ft. 2 in.

14. State a rule for finding the base when the area and altitude are given, also for finding the altitude when the area and base are given.

1. A plane figure bounded by straight lines is a **polygon**, meaning *many angles*.

2. A quadrilateral is a polygon of 4 sides.

3. You will seldom use any polygon except the triangle and quadrilateral. It may be well to know that a figure of 5 sides is a **pentagon**; of 6 sides, a **hexagon**; of 8 sides, an **octagon**; of 9 sides, a **nonagon**; of 10 sides, a **decagon.**

4. The quadrilaterals are:

The **square**, having 4 right angles and 4 equal sides.

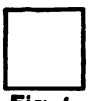
The **oblong**, having 4 right angles, and opposite sides equal. Squares and oblongs are also called **rectangles.**

The **rhombus**, having no right angles, but all its sides equal.

The **rhomboid**, having no right angles, and only opposite sides equal.

The **trapezoid**, having only two of its opposite sides parallel.

The **trapezium**, having none of its sides parallel.


Fig. 1.

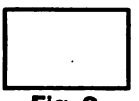

Fig. 2


Fig. 3


Fig. 4


Fig. 5


Fig. 6

5. Name each of these figures.

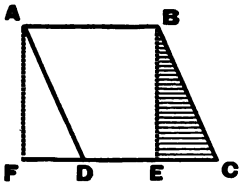
6. State the rule for finding the area of a rectangle.

7. State the rule for finding one dimension when the area and the other dimension are given.

8. What will be the cost of paving and curbing a street $\frac{3}{4}$ of a mile long and 42 ft. wide, if the paving costs \$1.25 a square yard, and the curbing costs 45¢ a linear foot?

9. A house is 32 ft. long and its rafters are 15 ft. long. Find the total weight of snow on this roof, if the weight on each square foot is 17 lb.

10. Measure the size of all the windows and find the amount of lighting surface in the room.



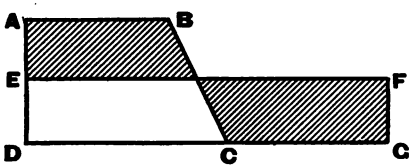
1. In this figure ABCD is a rhombus. If we cut off the shaded part BEC and place it on the other side it will cover the part marked AFD. We now have the figure ABEF.

2. What is the name of the figure ABEF? How do you find its area?

3. Compare the base and altitude of the rhombus ABCD with the base of the square, ABEF.

4. Do you see that the area of a rhombus is equal to that of a rectangle having the same base and altitude?

5. Make a drawing and see if the same rule is true of a rhomboid.



6. Give the name of the figure ABCD.

7. Cut out of paper a trapezoid four times as large as this figure.

8. Fold so that the two parallel edges will coincide. Crease and cut on the crease. Place the two pieces so that what were the two parallel sides shall form one continuous side. See EFGD. What form have you now?

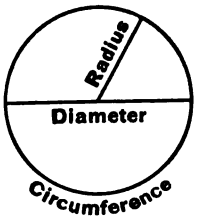
9. The base of this rectangle is the sum of what two sides of the trapezoid?

10. The altitude of the rectangle is what part of the altitude of the trapezoid?

Learn: To find the area of a trapezoid multiply the sum of the two parallel sides by $\frac{1}{2}$ of the altitude.

Find the area of trapezoids of the following dimensions:

	Altitude	Parallel Sides		Altitude	Parallel Sides
11.	60 ft.	11 ft., 16 ft.	12.	40 ft.	60 ft., 130 ft.
13.	6 ft.	13 ft., $6\frac{1}{2}$ ft.	14.	86 in.	120 in., 96 in.
15.	16 yd.	25 yd., 21 yd.	16.	$4\frac{1}{2}$ in.	25 in., 20 in.



1. The line that bounds this circle is called its ——.

2. The straight line that passes through the center touching both sides is the ——.

3. What is the radius?

4. The circumference of every circle is how many times its diameter?

5. This ratio is usually expressed by the Greek letter π (pi). $\pi = 3.1416$.

6. How do you find the circumference when you know the diameter?

7. How do you find the diameter when you know the circumference?

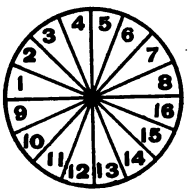


Fig. 1

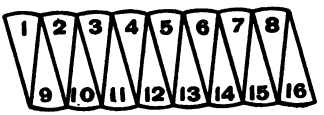


Fig. 2

8. Cut out of paper a circle whose diameter is about 6 in. Cut it into 16 parts as shown in Fig. 1.

9. Arrange the parts as shown in Fig. 2. Is the area of Fig. 2 the same as that of Fig. 1?

10. What is approximately the shape of Fig. 2? How do you find its area?

Learn: To find the area of a circle multiply one half of the circumference by the radius.

11. What dimensions of a circle do you need to know to find its area? If only one dimension is given how do you find the other?

Find the area of a circle when the:

12. Diameter is 17.5 ft.
13. Radius is $4\frac{1}{4}$ ft.
14. Circumference is 50 in.
15. Circumference is 5 ft. 4 in.
16. Radius is 20 rd.
17. Diameter is 28 ft.

1. All kinds of lumber are measured by board feet.

2. According to size and use pieces of lumber are called boards, planks, scantlings, joists, plates, girders, beams, etc.

3. A *board foot* is equivalent to a square piece of board 1′ long, 1′ wide, and 1″ thick. Boards less than 1″ in thickness are reckoned as one inch.

4. Find the number of board feet in a plank 18′ by 9′ by 3″.

$18' \times \frac{9}{12} \times 3 = \text{——}$.

If the plank was 1″ thick, each foot in length would contain $\frac{9}{12}$ of a board foot. Then 18′ would contain $18 \times \frac{9}{12}$; and since it is 3″ thick it will contain $(18 \times \frac{9}{12} \times 3)$ board feet. Use cancelation.

NOTE: — When boards are 1 in. thick the number of board feet in a board is the same as the square feet of one side.

5. How many feet of lumber are required for a tight board fence 140 ft. long and $5\frac{1}{3}$ ft. high?

6. If each plank is 2 in. thick, how many feet of lumber will be needed for a floor 12′ by 16′?

7. One end of a barn measures 40 ft. in width and 20 ft. to the eaves, and the altitude of the gable is 15 ft. How many feet of lumber will cover this end?

8. If each side of the roof measures 80 ft. by $25\frac{1}{2}$ ft., find the number of feet of boards that will cover the roof.

9. If the floor is covered with 3-in. planks find the cost of the floor at $18 per M.

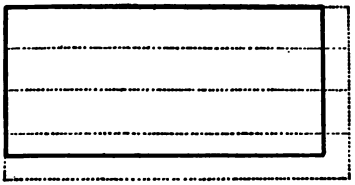
10. Find the cost of 15 joists, each $20' \times 9'' \times 6''$, at $1.85 per C.

11. How many board feet are there in 8 sticks of timber $18' \times 10'' \times 12''$?

12. At $16.73 per M, find the cost of 35 planks, each $24' \times 16'' \times 3''$.

13. How many boards 12 ft. long will it take to enclose a square field 48 rd. on a side, with a fence 5 boards high?

14. If each board is 6 in. wide, find the cost of the boards at $15 per M.



1. This diagram represents a room 18 ft. long, $10\frac{1}{2}$ ft. wide. How many breadths of carpeting 1 yard wide will be needed? How long will each breadth be if there is a loss of 6 in. on each breadth for matching? How many yards of carpet will it take for the room?

$10\frac{1}{2}$ ft. $\div$ 3 ft. $= 3\frac{1}{2}$ $\therefore$ 4 breadths.
18 ft. + 6 in. $= 18\frac{1}{2}$ ft. $= 6\frac{1}{6}$ yd. length.
$6\frac{1}{6}$ yd. $\times$ 4 $= 24\frac{2}{3}$ yd.

Since breadths of carpet are not split, it will be necessary to buy 4 breadths. Each breadth will be 6 in. longer than the room or $18\frac{1}{2}$ ft. long.

Learn: Divide the width of the room by the width of the carpet to find the number of breadths when the carpet runs lengthwise. Add the loss in matching to the length of the room, to find the length of each breadth. Multiply the length of each breadth by the number of breadths to find the amount of carpet needed.

2. How would you find the number of breadths needed when the carpet runs widthwise?

3. A room is 18 ft. by 24 ft. The carpet is $\frac{3}{4}$ of a yard wide and runs lengthwise. If there is a waste of 9 in. on each breadth for matching, how many yards of carpet will it take for the room? At \$1.25 a yard, find the cost of the carpet.

When the breadths run lengthwise, find the cost of carpeting from the following data:

	Room	Width of Carpet	Loss in Matching	Price a Yard
4.	12′ × 16′	1 yd.	8 in.	\$ .75
5.	15′ × 17′	$\frac{3}{4}$ yd.	9 in.	.65
6.	16′ × 21′	1 yd.	none	1.25
7.	18′ × 24′	27 in.	1 ft.	.85
8.	16′ × 19′	1 yd.	6 in.	.90
9.	17′ × 21′	$2\frac{1}{4}$ ft.	$\frac{1}{2}$ yd.	1.10

1. American wall paper is usually $1\frac{1}{2}$ ft. wide and 24 ft. long for a single roll and 48 ft. long for a double roll. A fraction of a roll is not sold and unless otherwise stated the price of a single roll is given. It is not easy to estimate exactly the amount of paper needed for a room, especially since paper hangers do not always use the same rule.

The following rule is sufficiently accurate for all purposes: Find the number of feet in the perimeter of the room, leaving out the widths of the doors and windows. Divide by $1\frac{1}{2}$ feet (the width of the paper), to find the number of strips required. Divide the length of a roll by the height of the room, to find the number of strips in a roll. Divide the strips needed for the room by the strips in a roll, to find the rolls required. In the first and third divisions, if there is a fraction take the next higher integer; in the second division take the next lower integer.

2. A hall is 18 ft. by $6\frac{1}{2}$ ft. and $7\frac{1}{2}$ ft. high. There is one door and one window, each $3\frac{1}{2}$ feet wide. At 24¢ a roll, find the cost of the paper.

$2\ (18\text{ ft.} + 6\frac{1}{2}\text{ ft.}) = 49\text{ ft. perimeter.}$
$49\text{ ft.} - (3\frac{1}{2}\text{ ft.} + 3\frac{1}{2}\text{ ft.}) = 42\text{ ft.}$
$42\text{ ft.} \div 1\frac{1}{2}\text{ ft.} = 28.$ No. of strips for room.
$24\text{ ft.} \div 7\frac{1}{2}\text{ ft.} = 3\frac{1}{5}$ ∴ 3 strips in a roll.
$28\text{ strips} \div 3\text{ strips} = 9\frac{1}{3}$ ∴ 10 rolls for room.
$24¢ \times 10 = \$2.40.$

NOTE — When the ceiling is to be papered find the quantity needed in the same way.

Find the cost of papering, including the ceiling:

	Rooms	Doors	Windows	Cost per Roll
3.	$20' \times 15' \times 9'$	$2, 4' \times 7'$	$2, 3\frac{1}{2}' \times 6'$	20¢
4.	$16' \times 12' \times 8'$	$1, 4' \times 6\frac{1}{2}'$	$2, 3' \times 6'$	28¢
5.	$15' \times 11' \times 8\frac{1}{2}'$	$3, 4' \times 7'$	$2, 3' \times 5\frac{1}{2}'$	30¢
6.	$30' \times 24' \times 12'$	$5, 4\frac{1}{2}' \times 7'$	$4, 3\frac{1}{2}' \times 6'$	24¢
7.	$18' \times 21' \times 10'$	$4, 4' \times 7\frac{1}{2}'$	$3, 3' \times 5\frac{1}{2}'$	18¢
8.	$16' \times 14' \times 9\frac{1}{2}'$	$3, 4\frac{1}{2}' \times 7'$	$2, 3\frac{1}{2}' \times 6'$	30¢

1. How many bushels of apples will a bin contain that is 30′ long, 8′ wide, and 9′ high?

(30 × 8 × 9) cu. ft. = 2160 cu. ft.
2160 cu. ft. ÷ $1\frac{1}{2}$ cu. ft. = 1440 ∴ 1440 bu.
} or 30 × 8 × 9 × $\frac{2}{3}$ bu. = 1440 bu.

2. A bin 24 ft. long, 6 ft. wide, and 2 ft. deep is $62\frac{1}{2}\%$ full of oats. How many bushels are in the bin?

$\frac{5}{8}$ × (24 × 6 × 2) cu. ft. = 180 cu. ft.
180 cu. ft. ÷ $1\frac{1}{4}$ cu. ft. = 144 ∴ 144 bu.
} or $\frac{5}{8}$ × 24 × 6 × 2 × $\frac{4}{5}$ bu. = 144 bu.

3. In measuring grain, etc., the even bushel is used. This contains 2150.4 cu. in., or approximately $1\frac{1}{4}$ cubic feet. In measuring bulky fruits the heaped bushel is used. This contains 2688 cu. in. or approximately $1\frac{1}{2}$ cu. ft. Hence 1 cu. ft. equals $\frac{4}{5}$ bu. or $\frac{2}{3}$ bu., depending upon the article measured.

Learn: a. For accurate purposes divide the number of cubic inches in the bin by 2150.4 *cu. in. when using the even bushel, and by* 2688 *cu. in. when using the heaped bushel.*

b. For practical purposes take $\frac{4}{5}$ *or* $\frac{2}{3}$ *of the number of cubic feet in the bin for the required number of bushels.*

4. A bin 6 ft. wide and 10 ft. long holds 150 bu. potatoes. How high is the bin?

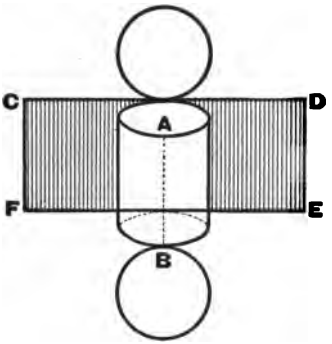
$150 \times 1\frac{1}{2}$ cu. ft. = 225 cu. ft.
$10 \times 6 \times x$ = 225 cu. ft.

5. A bin is 6 ft. wide and $4\frac{1}{2}$ ft. deep. If it contains 324 bu. of potatoes, how long is it?

Find the number of cubic feet in a bin that holds :

6. 600 bu. of wheat
7. 400 bu. of apples.
8. 400 bu. of oats.
9. 1200 bu. of potatoes.
10. 900 bu. of shelled corn.
11. 1500 bu. of onions.

12. At $1.05 a bushel, find the value of the wheat that fills $\frac{2}{3}$ of a bin $12\frac{1}{2}' \times 6 \times 9'$.



1. Make a paper cylinder and cut it on the line AB as in Fig. 1. Open to a flat surface. What form have you?

2. The rectangle CDEF formed the curved surface of the cylinder. The curved surface is usually called the **convex surface.**

3. The two circles formed what part of the cylinder?

Learn: To find the entire surface of a cylinder:

a. Multiply the circumference by the height to find the convex surface.

b. To the convex surface add the areas of the two ends.

Find the entire surface of cylinders having the following dimensions, calling π $3\frac{1}{7}$.

4. Circumference 88 ft., height 24 ft.

5. Circumference 132 ft., height 30 ft.

6. Diameter 28 ft., height 40 ft.

7. Find the volume of a cylinder whose diameter is 21 in. and height 14 in. How many gallons will this cylinder hold?

$3\frac{1}{7} \times 21$ in. $= 66$ in. cir.
$(10\frac{1}{2} \times 33)$ sq. in. = area of base.
$(10\frac{1}{2} \times 33 \times 14)$ cu. in. = volume.
$10\frac{1}{2} \times 33 \times 14 \times 7\frac{1}{2}$ gal. = gal.

First find the area of the base. Multiply this by the height to find the volume. Since there are $7\frac{1}{2}$ gal. in a cubic foot multiply $7\frac{1}{2}$ gal. by the number of cubic feet in the volume.

Learn: To find the volume of a cylinder multiply the area of the base by the height.

Find the capacity in gallons of cisterns of the following dimensions, calling π 3.1416.

8. Diameter 6 ft., height 6 ft.

9. Circumference 25 ft., height 15 ft.

1. How many bricks will be required to build a wall 84 ft. long, 32 ft. high, and 1 ft. thick? 22 bricks make 1 cu. ft. of wall.

2. How many 3-in. cubes can be cut from a block 27 in. long, 12 in. wide, and 9 in. thick, if no allowance is made for waste in cutting?

3. What is the area in acres of a triangle whose base is 56 rd., and altitude is 64 rd?

4. At \$45 an acre find the value of a piece of land whose parallel sides are 40 rd. and 30 rd. and whose altitude is 60 rd.

5. A tank 9 ft. deep, 14 ft. long, and $7\frac{1}{2}$ ft. wide is $\frac{2}{3}$ full of water. At the rate of 5 gal. a minute, how long will it take to pump out the water?

6. At \$72 an acre a farm is worth \$12,240. What will it cost to fence it at \$1.25 a rod, it if is in the form of a rectangle 160 rd. wide?

7. A bin is 8.5 ft. long, 4.25 ft. wide, and 3.75 ft. deep. How many bushels of oats will it hold?

8. At \$4.25 a cord, what is the value of a pile of wood 72 ft. long, 4 ft. wide, and 12 ft. high? Find the number of cords in the pile without finding the number of cubic feet.

9. At \$18 per M, find the cost of planks for flooring a barn 40 ft, by 32 ft., if each plank is 16 ft. long, 15 in. wide, and 2 in. thick.

10. What is the circumference of the largest circle that can be drawn on a 9-ft. square?

11. The area of a triangle is 36 sq. yd., and the base is 36 ft. What is the altitude?

12. If a bin is 8 ft. square, how deep must it be to hold 256 bu. of apples?

13. At 15¢ a square rod, what will it cost to cover with gravel a driveway $49\frac{1}{2}$ ft. wide round the outside of a circular park 36 rd. in diameter?

NOTE: — In measuring cellars and buildings in estimating for bricks or stone work, masons measure the outside of the walls, thus counting each corner twice. This is considered fair because of the greater labor in constructing the corners. For the same reason no allowance is made for an opening unless it is a large one, when one half of the opening is deducted.

1. Estimate the number of bricks required for a church $120' \times 85' \times 30'$. The walls are 2 ft. thick, and there are 12 windows, each 3 ft. by 6 ft. and 4 doors, each 5 ft. by 10 ft.

$2\ (120 + 75)$ ft. $= 390$ ft. length.
$(390 \times 30 \times 2)$ cu. ft. $= 23{,}400$ cu. ft.
$\frac{1}{2}\ (12 \times 3 \times 6 \times 2)$ cu. ft. $= 216$ cu. ft. in windows.
$\frac{1}{2}\ (4 \times 5 \times 10 \times 2)$ cu. ft. $= 200$ cu. ft. in doors.
$\{23{,}400 - (216 + 200)\} \times 22$ bricks $=$ ——.

2. Estimate the quantity of bricks required for a house $48' \times 25' \times 21'$. The walls are 1 ft. thick, and 480 cu. ft. are allowed for openings.

3. A room is 30 ft. long, $22\frac{1}{2}$ ft. wide, and 10 ft. 8 in. high. Making no allowance for openings find the cost for papering the walls and ceiling at 75¢ a roll, and of molding at 10¢ a foot.

4. What is the value of a lot of land 75 ft. deep, 50 ft. wide in front and 42 wide in the rear, at 65¢ a square foot?

5. If $\frac{5}{11}$ of an acre of land cost \$37.75, what will $43\frac{1}{5}$ acres cost?

6. A man owned $\frac{5}{8}$ of an acre of land. How much had he left, after selling $28\frac{3}{4}$ square rods?

7. A man had a circular pond 30 yd. in diameter. He made a circular walk round it 6 ft. wide. Find the area of the walk.

8. How many square feet of zinc will be required to line completely a cylinder 8 ft. in diameter and 40 ft. high?

9. At 55¢ a cubic yard, what will it cost to dig a well 5 ft. in diameter and 30 ft. deep?

10. Find the number of board feet in 14 planks, 15 ft. long, 16 in. wide, and 3 in. thick.

1. At 6% a year find the interest on \$1000 for 48 da.

2. A farmer sold a horse for \$105 and gained $16\frac{2}{3}\%$. What did the horse cost?

3. A teacher said, "I should have 50 pupils in my room, but 5% are absent." How do you know she made a mistake?

4. At \$1 per C, find the cost of 10 boards, each 12 ft. long, and 10 in. wide.

5. When a number is used twice as a factor, that is, is multiplied by itself, the product is called the **square** of the number.

6. Name the squares of 3, 5, 6, 8, and 12.

7. Use 4 three times as a factor. $4 \times 4 \times 4 =$ ——.

8. When a number is used three times as a factor the product is called the **cube** of the number.

9. Name the cubes of 2, 5, 3, 8, and 10.

10. At \$13.50 each, how much will 100 trunks cost?

11. At \$2.50 each, how many chairs can be bought for \$50?

In the following examples give the answer in terms of both the minuend and subtrahend:

12. 3 lb. − 32 oz.
13. 4 hr. − 180 min.
14. 4 yr. − 36 mo.
15. 5 yd. − 12 ft.
16. 5 ft. − 48 in.
17. 9 ft. − 2 yd.
18. \$12 − 350¢
19. 2 T − 2000 lb.
20. 5 dimes − 20 ct.
21. 4 bu. − 8 pk.
22. 6 pk. − 24 qt.
23. 10 qt. − 8 pt.

24. *Find the missing terms :*

Minuend	45	x	43	53	x	61	x	95	74
Subtrahend	x	36	27	x	34	29	64	x	39
	24	15	x	11	42	x	19	32	x

1. If a number is multiplied by 8 the product is 480. What is the number?

2. Find the interest on \$600 from Aug. 16, to Nov. 24, at $7\frac{1}{2}\%$.

3. What will 8 lb. 12 oz. of butter cost if $1\frac{1}{2}$ lb. cost 54 cents?

4. What is a mixed number? Write one. Change it to some other form without changing its value.

5. A rectangular garden is 200 ft. long and 150 ft. wide. Find the area of a walk 3 ft. wide running all round it and also through the center in both directions.

6. If it requires 440 ft. of lumber to cover the gable ends of a barn 40 ft. wide, what is the height of the gable?

7. How long a band of iron will it take to surround a cylindrical tank 15 ft. 8 in. in diameter?

8. There are 360 persons in a room 60 ft. long by 40 ft. wide. To allow 100 cu. ft. of air to each person, how high ought the room to be?

9. At 27¢ a square yard find the cost of plastering a room 22 ft. 3 in. long, 16 ft. wide, and 12 ft. high. There are 2 windows, each 6′ by $3\frac{1}{2}$′; 4 doors, each 7′ by 4′, and a baseboard 9 in. high.

10. A tank can be filled by one pipe in 15 min., and by another in 30 min. In what time can it be filled by both together?

11. Suppose water runs in through the first pipe and out through the other. In what time then will the tank be filled?

12. What will 25 bu. 3 pk. 2 qt. of cherries cost at 8¢ a quart?

13. A room is 16 ft. 4 in. by 15 ft. How many yards of carpet, $\frac{3}{4}$ yd. wide, will it take for the room, if the breadths run widthwise and there is a waste of 6 in. on each breadth for matching?

1. **Percentage** is the name given to that part of arithmetic where all reckoning is done by hundredths.

2. Percentage is therefore an application of decimal fractions.

3. 7% of anything is how many hundredths of it?

4. There are three problems in percentage:
   a. To find a given percentage of a number.
   b. To find what per cent one number is of another.
   c. To find a number when a percentage of it is given.

5. Find 5% of \$80.

Solution A.
$$\begin{array}{r} \$80 \\ .05 \\ \hline \$4.00 \end{array}$$

B. $5\% = \frac{1}{20}$

$\frac{1}{20}$ of \$80 = \$4.

Learn: To find a percentage of a number, multiply the number by the rate per cent expressed either decimally or as a common fraction.

6. \$150 is what per cent of \$9000?

A. $\frac{150}{9000} = \frac{1}{60}$

$\frac{1}{60}$ of $100\% = 1\frac{2}{3}\%$

B. $9000\overline{)150.00}$ $.01\frac{2}{3}$

$\begin{array}{r} 9000 \\ \hline 6000 \\ 9000 \\ \hline \end{array}$

$\frac{6000}{9000} = \frac{2}{3}$

Learn: To find what per cent one number is of another, first find what part it is and change that fractional part to hundredths.

7. \$240 is 8% of what sum?

A. $8\% = \$240$

$1\% = \frac{1}{8}$ of $\$240 = \$30$

$100\% = 100 \times \$30 = \$3000$

B. $.08\overline{)\$240.00}$ \$3000.

Learn: To find a number when a percentage of it is given, divide by the given rate to find 1%, then multiply by 100, or divide the given percentage by the rate per cent expressed decimally.

1. The number of which a certain number of hundredths is to be taken is called the **base.**

2. The number of hundredths that we take of the base is called the **rate** or the **rate per cent.**

3. The result obtained by taking a certain per cent of the base is called the **percentage.**

4. Find 5% of \$300.

\$300 Base
.05 Rate
\$15.00 Percentage

The percentage is the product of the base and the rate. You can see that the base is the multiplicand, the rate is the multiplier, and the percentage is the product.

5. Fill in the blanks in the following:
   a. Multiply the base by the rate to find the ——.
   b. Divide the percentage by the rate to find the ——.
   c. Divide the percentage by the base to find the ——.

Find the missing term in the following:

	Base	Rate	Percentage		Base	Rate	Percentage
6.	\$70	4%	x	7.	x	2%	\$40
8.	\$850	x	\$425	9.	\$8.20	105%	x
10.	x	3%	\$7.50	11.	\$108	x	\$5.40

12. The increase in population in one year in a certain town was 5000, which was $15\frac{5}{8}\%$ of the population. What was the population at the end of the year?

13. A man bought an automobile for \$1250, and sold it at a loss of \$500. Find the per cent of loss?

14. A farmer cut 339 tons of hay this year. If this is an increase of 13% over what he cut last year, how many tons did he cut last year?

15. For the month of December, James received a mark in spelling of 87%. If he missed 52 words during the month, how many did he spell correctly?

16. During January, James was given 400 words to spell. If he spelled 368 correctly, what is his per cent mark for the month?

1. The excess of the selling price over the cost is **profit.**

2. The excess of the cost over the selling price is **loss.**

3. The selling price is always the cost plus the profit or the cost less the loss.

4. The gain or loss per cent is always reckoned on the cost.

5. For how much must a merchant sell goods that cost \$450, so as to gain 10%?

$\frac{1}{10}$ of \$450 = \$45.
\$450 + \$45 = \$495.

10% = $\frac{1}{10}$. $\frac{1}{10}$ of \$450 = \$45, the gain. \$450 + \$45 = \$495, the selling price.

6. Some goods were marked to be sold for \$60, but being damaged were sold at a loss of 15%. Find the selling price.

15% of \$60 = \$9, the loss.
\$60 − \$9 = \$51, the selling price.

7. A boy sold his bicycle for \$81 at a loss of 10%. Find the cost of the bicycle.

100% − 10% = 90%, the selling per cent.

90% = \$81.
1% = $\frac{1}{90}$ of \$81 = \$.90
100% = 100 × \$.90 = \$90

or

$$90\overline{)8100.}\quad \$90$$

8. A dealer bought some goods for \$640 and sold them for \$716.80. What was his gain per cent?

\$716.80 − \$640 = \$76.80 gain
\$76.80 ÷ \$640 = .12 ∴ 12%

First find the gain, then find what per cent the gain is of the cost.

9. A man bought a lot for \$850 and sold it at a gain of 25%. If he sold it at 15% less than he asked for it, find his asking price.

\$850 + $\frac{1}{4}$ of \$850 = \$1062.50, the selling price.

85% = \$1062.50.
100% = \$1250.

If he sold it for 15% less than he asked for it, the selling price must be 85% of the asking price.

NOTE: — These are typical problems and should be studied carefully.

1. A dealer lost 15% by selling eggs at 17¢ a dozen. Find the cost of a dozen.

2. A merchant's income is $5730 this year. This is a gain of $18\frac{3}{4}\%$ on the capital invested. His income last year was 25% of the capital. Find his income last year.

3. If goods that cost $.84 a yard are sold at $.63, what is the loss per cent?

4. A lady spent $64.50 for jewelry and dress goods, paying 15% more for dress goods than jewelry. How much did she pay for each?

Let 100% = money paid for jewelry
and 115% = money paid for dress goods
then 215% = $64.50.

5. A and B together have $1680, and A has 25% less money than B. How much money has each?

6. Find the cost of a ton of hay which was sold at $15 at a gain of 20%.

7. What would have been the gain per cent if the hay in example 6 had been sold at $13.50 a ton?

8. A earns 18% more in a week than B. If the sum of their wages is $76.30, how much does each earn?

9. A man spent $62\frac{1}{2}\%$ of the money he withdrew from a bank for an automobile that cost $1250. How much did he withdraw? If he withdrew $33\frac{1}{3}\%$ of all he had on deposit, what was the original deposit?

10. When the selling price was $63, the rate of loss was $12\frac{1}{2}\%$. Find the cost.

11. A merchant bought 960 yd. of silk at $2.00 a yard. He sold $\frac{1}{4}$ of the silk at a gain of 25%, $\frac{1}{3}$ of it at a gain of 20%, and the remainder at a loss of 15%. For what was the silk sold?

12. A man sold 5000 bu. of wheat for $300 less than it cost him, and thus lost 12%. What did the wheat cost him a bushel?

1. Mr. Walker owns a house valued at \$6000. He insures it for $\frac{3}{4}$ of its value in the Home Fire Insurance Co. This means that the company promise to pay him \$4500 if the house is burned, within 5 years.

2. For this promise Mr. Walker pays the company $1\frac{1}{2}\%$ of the amount for which the property is insured. Find $1\frac{1}{2}\%$ of \$4500.

3. Security against loss or damage of any kind is called **insurance.**

4. There are several kinds of insurance. The more common forms are **fire insurance, marine insurance, accident insurance** and **life insurance.** The first insures against any loss by fire, the second against any loss at sea, the third against any loss from an accident, and the fourth against loss of life.

5. The written agreement between the company and the person insured is called a **policy.**

6. The sum paid for the insurance is the **premium.** The premium is a certain per cent of the amount named in the policy.

7. The insurer is often called the **underwriter.**

8. Property is usually insured by the underwriters for less than its full value.

ORAL EXERCISES

9. A house valued at \$8000 was insured for $\frac{3}{4}$ of its value at $1\frac{1}{2}\%$. What was the premium?

Find the premium in the following:

	Value	Part Insured	Rate		Value	Part Insured	Rate
10.	\$600	$\frac{5}{6}$	3%	11.	\$1600	$\frac{3}{4}$	2%
12.	\$800	$\frac{3}{4}$	2%	13.	\$10,000	$\frac{3}{4}$	1%
14.	\$1500	$\frac{4}{5}$	$1\frac{1}{2}\%$	15.	\$5000	$\frac{4}{5}$	$1\frac{1}{2}\%$
16.	\$2000	$\frac{3}{4}$	1%	17.	\$9000	$\frac{2}{3}$	1%
18.	\$2400	$\frac{2}{3}$	1%	19.	\$12,000	$\frac{5}{6}$	$1\frac{1}{2}\%$

1. A building worth \$64,000 was insured for $\frac{3}{4}$ of its value at $1\frac{1}{2}\%$. Find the premium.

$\frac{3}{4}$ of \$64,000 = \$48,000
$1\frac{1}{2}\%$ of \$48,000 = \$720

Since the building was insured for only $\frac{3}{4}$ of its value, we first find $\frac{3}{4}$ of \$64,000, which is \$48,000. We then find $1\frac{1}{2}\%$ of \$48,000, which is \$720.

2. A man insured his house for $\frac{2}{3}$ of its value at $1\frac{1}{4}\%$, and paid a premium of \$45. Find the value of the house.

\$45 ÷ .0125 = \$3600
$\frac{2}{3}$ of value = \$3600
$\frac{3}{3}$ of value = \$3600 × $\frac{3}{2}$ = \$5400

First divide the premium by the rate to find the sum insured, which is \$3600. Since the house was insured for $\frac{2}{3}$ of its value, \$3600 = $\frac{2}{3}$ of the value, and $\frac{3}{3}$ of the value must be \$5400.

3. A factory valued at \$75,000 was insured for $\frac{3}{5}$ of its value. If the premium was \$750, what was the rate of insurance?

$\frac{3}{5}$ of \$75,000 = \$45,000
\$750 ÷ \$45,000 = .01$\frac{2}{3}$

Find the sum for which the factory was insured, which is \$45,000. Then divide the premium by the sum insured to find the rate of insurance.

4. A house worth \$5600 is insured for $\frac{7}{8}$ of its value at 1%. Find the premium.

5. An insurance company charged \$120 for insuring a house for \$8000. What was the rate of insurance?

6. The premium at $1\frac{1}{2}\%$ for insuring a house was \$75. For what sum was the house insured? If it was insured for $\frac{4}{5}$ of its value, what was the value of the house?

7. If the cost of insuring property at $2\frac{1}{4}\%$ is \$364.50, what is the value of the property?

8. An insurance company asked \$120 as premium on property insured for \$16,000. What was the rate of insurance?

9. Find the cost of insuring some goods valued at \$23,425, if the risk was taken at $1\frac{1}{4}\%$ on $\frac{3}{5}$ of the value.

10. When the premium is \$250 and the rate of insurance is $2\frac{1}{2}\%$, find the face of the policy. If the property is insured for $\frac{3}{4}$ of its value, find the whole value.

1. A ship worth \$52,000 was insured for $\frac{3}{4}$ of its value at $2\frac{1}{2}\%$. The cargo worth \$8640 was insured for $\frac{2}{3}$ of its value at 3%. Find the whole premium.

2. A house worth \$12,000 is insured for $\frac{2}{3}$ of its value. Find the premium, if the rate is $1\frac{3}{5}\%$.

3. Mr. J. paid \$250 for insurance on his stock of goods. If the face of the policy was \$10,000, what was the rate of insurance?

4. When the premium is \$350 and the rate of insurance is 5%, find the the face of the policy. If the property is insured for $\frac{3}{4}$ of its value, find the whole value.

5. How large an insurance can a man place on his house by paying a premium of \$122.50, if the rate of insurance is $1\frac{3}{4}\%$?

6. A man insures his life for \$2500, at the rate of \$22.50 for every \$1000. What is his annual premium?

7. A man insured his life for \$3000, paying \$24.25 semi-annually for every \$1000. Find the total amount paid in premiums in 12 years. At his death his wife received \$3000. How much more did she receive than had been paid in premiums?

8. A factory valued at \$50,000 is insured for $\frac{3}{5}$ of its value. If the premium is \$500, what is the rate of insurance?

9. If a merchant pays \$150 for insuring \$8000 worth of goods, what is the rate of insurance?

10. \$206.25 was the premium paid for insuring a factory at $1\frac{3}{8}\%$. Find the face of the policy.

11. A man insured his barn for $\frac{3}{4}$ of its value at $1\frac{1}{4}\%$, and paid a premium of \$15. Find the value of the barn.

12. A man paid \$87 for insuring his house worth \$7250. What was the rate of insurance?

13. A house worth \$3500 is insured for $\frac{3}{5}$ of its value at $1\frac{3}{4}\%$. Find the premium.

1. Mr. Snow sold $1200 worth of goods for Mr. Ryan, and asked 2% for making the sale. How much did Mr. Snow receive? After paying Mr. Snow how much did Mr. Ryan receive for his goods?

2. A person who transacts business for another is called an **agent.** Agents are known by different names according to the work they do, as, a **broker,** a **collector,** a **commission merchant.**

3. The money paid to the agent for his work is called **commission** or **brokerage.**

4. Agents receive money for their employers by collecting debts or selling property and their commission is some per cent of the money received.

5. Agents also expend money for their employers and then their commission is some per cent of the money paid out.

6. The money left after the commission and all other expenses have been paid is called the **net proceeds.**

7. The entire cost of a purchase is the price paid plus the commission and all other expenses.

ORAL EXERCISES

Find the commission on the following sales at specified rates:

8. $800, 2%.	9. $700, 3%.	10. $1500, $3\frac{1}{2}$%.
11. $1000, 5%.	12. $4768, 1%.	13. $2000, $2\frac{1}{4}$%.
14. $200, 20%.	15. $3200, $1\frac{1}{2}$%.	16. $6000, $1\frac{1}{2}$%.
17. $900, $1\frac{1}{2}$%.	18. $1800, 2%.	19. $10,000, 25%.

20. An agent sold $4000 worth of goods at $1\frac{3}{4}$% commission. Find the commission.

21. A collector's commission at 3% for collecting a debt was $60. How much did he collect?

22. An agent charged $100 for selling a farm for $2000. What was the rate of his commission?

1. A commission merchant sold 150 barrels of apples at \$4 a barrel at a commission of 2%. Find his commission.

\$4 × 150 = \$600
2% of \$600 = \$12

Find the selling price of 150 barrels, which is \$600. Since the agent receives a commission on the money received for selling goods, we find 2% of \$600.

2. An agent bought for me 300 bbl. of flour at \$4.75 a barrel. If his commission was 2% and other charges \$31.50, what was the entire cost to me?

\$4.75 × 300 = \$1425
2% of \$1425 = \$28.50
\$28.50 + \$31.50 = \$60
\$1425 + \$60 = \$1485

Find the cost, which is \$1425. 2% of \$1425 = \$28.50, the commission. \$28.50 + \$31.50 = \$60, the amount that must be paid in addition to the cost. \$1425 + \$60 = \$1485.

3. A real estate agent sold a farm for \$4000, and charged a commission of 3% and \$12 for other expenses. Find the net proceeds.

3% of \$4000 = \$120
\$120 + \$12 = \$132
\$4000 − \$132 = \$3868

Find the commission or \$120. \$120+\$12 = \$132, the total expenses. The selling price, \$4000, less the total expenses, \$132, equals the net proceeds, \$3868.

4. An agent's commission at 3% for selling houses was \$810. If he sold 3 houses, find the selling price of each house.

\$810 ÷ .03 = \$27,000
\$27,000 ÷ 3 = \$9000

Divide the commission by the rate of commission to find the selling price, \$27,000. If 3 houses sold for \$27,000, each house must sell for $\frac{1}{3}$ of \$27,000 or \$9000.

5. An agent sent his employer \$23,226.30 as the net proceeds of a sale and kept \$353.70 as his commission. What was the rate of his commission?

\$23,226.30 + \$353.70 = \$23,580
\$353.70 ÷ \$23,580 = .015

Add the net proceeds and the commission to find the selling price. Divide the commission by the selling price to find the rate of commission.

6. An agent sold goods for a merchant to the amount of \$1200 and charged a commission of $2\frac{1}{2}$%. Find the commission and the net proceeds.

1. An agent bought 70 bbl. of flour at \$5.25 a barrel. His commission was 3%. Find his commission. Find the total cost to the merchant.

2. A commission merchant sold a lot of goods for \$1480, charging $2\frac{1}{2}$ commission. Find the commission. What sum should he remit (send back) to his employer?

3. A broker bought some goods for a merchant for which he paid \$2146. His commission for buying was $\frac{1}{2}$%. What did the goods cost the merchant?

4. A collector charged 3% for collecting bills. In one month he received \$345. How much did he collect?

5. An agent sold 23 wagons at \$85 each on 6% commission. Find the net proceeds.

6. Mr. A. received \$7990 as the net proceeds for his farm sold by an agent on 6% commission. Find the selling price of the farm.

7. A collector charged a commission of $3\frac{1}{2}$%. If he collected 95% of a debt of \$2148, what sum should he send his employer?

8. What will be the net proceeds of a sale of 525 bbl. of beef at \$18.25 a barrel, allowing 3% commission and 5¢ a barrel for storing?

9. An agent's commission at $4\frac{1}{2}$% for selling horses was \$270. If he sold 40 horses, find the selling price of each horse.

10. An agent sent his employer \$7742.10 as the net proceeds of a sale and kept \$117.90 as his commission. What was the rate of his commission?

11. Keeping 5% for his services, an agent remitted \$237.50 to his employer. For how much did he sell the goods?

12. A real estate dealer sold 75 lots of land at \$275 a lot. If he charged 2% commission for selling and \$5 a lot for recording the deed, what will be the net proceeds of the sale?

In their catalogue the publishers mark the price of the Standard Dictionary as $25.00, but allow a reduction of 10% from this price for cash. How much is the reduction? What is the price of the dictionary after the reduction?

1. A reduction from the marked or list price of an article is called **commercial discount** or **trade discount.**

2. Manufacturers and dealers issue price lists of their goods. Such a list is called the **list price.** The price remaining after all discounts have been taken away is the **net price** or **net amount.**

3. Frequently in trade several discounts are made which vary as the condition of the market varies.

4. What is due on a bill of $500, subject to a discount of 20% and 10%?

A.		B.	
100% − 20	= 80%.	20% of $500	= $100.
10% of 80%	= 8%.	$500 − $100	= $400.
80% − 8%	= 72%.	10% of $400	= $40.
72% of $500	= $360.	$400 − $40	= $360.

NOTE: — A study of B will show you that the first discount is taken from the list price, the second discount is taken from the remainder, and so on.

5. Find the net amount of a bill of $1440 with 25%, 10%, and 5% off.

Find the net amount of a bill of:

6. $1920 with 25% and $7\frac{1}{2}$% off.

7. $1275 with 20% and 15% off.

8. $1088 with 20%, 10%, and 5% off.

9. Find the net price of 480 lb. of tea at $62\frac{1}{2}$¢ a pound, with $37\frac{1}{2}$% and 15% off.

10. Find the net amount of a bill of $1285, if the discounts are $\frac{1}{5}$ and 5%.

11. On a bill of $1826, discounts of 40% and 10%, or 30% and 20% were offered. Which discount should you accept? How much would you save?

1. A bill of goods at list price amounted to \$750. If the net price was \$625, what was the per cent of discount?

2. After a discount of 15% the net amount of a bill was \$552.50. What was the face of the bill?

3. Find a single discount equivalent to discounts of 15% and 10%.

$$\begin{aligned} 100\% - 15\% &= 85\%. \\ 10\% \text{ of } 85\% &= 8\tfrac{1}{2}\%. \\ 15\% + 8\tfrac{1}{2}\% &= 23\tfrac{1}{2}\%. \end{aligned}$$

4. Find a single discount equivalent to discounts of 45% and 10%.

5. The list price of a bill of goods amounted to \$1672. If the discounts were 30%, 10%, and 5%, find the net price.

6. What is the marked price of goods sold for \$94.50 after discounts of 30% and 10% have been deducted?

A.
$$\begin{aligned} 100\% - 30\% &= 70\%. \\ \$94.50 \div .70 &= \$135. \\ 100\% - 10\% &= 90\%. \\ \$135 \div .90 &= \$150. \end{aligned}$$

B.
$$\begin{aligned} 100\% - 30\% &= 70\%. \\ 10\% \text{ of } 70\% &= 7\%. \\ 70\% - 7\% &= 63\%. \\ \$94.50 \div .63 &= \$150. \end{aligned}$$

7. Find the marked price of a piano sold for \$135 after discounts of $16\frac{2}{3}\%$ and 10% have been deducted.

8. Find the marked price of goods sold for \$98 after discounts of $12\frac{1}{2}\%$ and $12\frac{1}{2}\%$ have been deducted.

9. A dealer bought a chair, the list price of which was \$3.50 at discounts of 20% and 5%. If he sold it at 14% above the list price, how much did he gain?

10. Deduct 20% and 5% from the following bill:

25 lb. crushed sugar, at 10¢; 40 lb. maple sugar at 12¢; 6 lb. cheese, at 13¢; 8 lb. butter, at 28¢; 4 lb. raisins, at 13¢; 2 lb. cream tartar, at 35¢. What is the net amount of the bill?

11. What is the retail price of a dictionary if it cost \$18 after a discount of 10% had been allowed?

1. A merchant bought a chair for \$8 at 20% discount, and sold it for \$10, at 15% and 6% off. How much did he gain?

2. A man bought $67\frac{1}{3}$ yd. of carpet at $\$1.87\frac{1}{2}$ at 15% discount. Find the net amount of his bill.

3. The gross amount of a bill is \$640 and the discounts are 10%, 10%, and 5%. Find the net amount of the bill.

4. Two merchants offer the same quality of goods at the same list price. A offers a discount of 10% and 5%, and B offers a discount of 15%. Of which merchant will it be more advantageous to buy? How much can be saved on a bill the list price of which is \$1050?

5. A dealer obtained \$360 for a piano on the list price of which he had discounted 50%. If he still made a profit of 20%, find the list price and the cost to the dealer.

6. A merchant asked for a suit of clothes 20% more than it cost him, but sold it for 80% of the asking price. What per cent did he lose?

7. A merchant bought some cloth at 25% below the list price and sold it at 20% above the list price. What per cent did he gain? If the list price was \$3.60 a yard, for how much was the cloth sold? How much did the merchant pay for it?

8. If goods are bought at 20% below the list price, with 5% off for cash, and sold at 14% above the list price, what per cent is gained?

9. A merchant bought some goods at discounts of 20%, 10%, and 5%. At what discount from the list price must he sell them to gain 25%?

10. Find a single discount equivalent to discounts of 30% and 10%.

11. What is the marked price of goods sold for \$283.50, after discounts of 30% and 10% have been deducted?

12. Find the trade discount on a bill of goods for \$2920 with 15% and 10% off.

1. By selling a piece of land for \$1600, a man lost 25%. Find the cost of the land. At what price should the land have been sold to have made 20%?

2. A man bought a house for \$2250 and sold it for \$2700. What per cent did he gain?

3. A house was insured for $\frac{3}{4}$ of its value at $\frac{3}{4}$%. If the premium was \$13.50, what was the value of the house?

4. An agent collected 80% of a debt of \$5600 and charged $4\frac{1}{2}$% commission. How much ought he to return to his employer?

5. A man paid \$25 for an insurance policy on his house. If the rate was $\frac{1}{2}$%, for how much was the house insured?

6. Find the interest on \$769.74 for 3 yr. 9 mo. at 4%.

7. An ocean steamer cost \$1,550,000. What premium must be paid to insure it at $2\frac{1}{4}$%?

8. The selling price of an article was \$73.50 after discounts of 30% and 30% had been made. What was the list price?

9. A commission merchant sold 500 pieces of cloth for \$130 a piece, and paid his employer \$55,250. What was his commission? What was the rate of his commission?

10. Find the cost of insuring a house for \$4000 at 60¢ on a thousand, and furniture for \$12,000 at 80¢ on a thousand, less a discount of 15% on each premuim.

11. A grain elevator in Minneapolis is valued at \$24,000 and the grain in it at \$25,000. The elevator is insured for $\frac{5}{6}$ of its value at $\frac{3}{4}$% and the grain is insured for $\frac{4}{5}$ of its value at $\frac{1}{2}$%. Find the entire premium.

12. A merchant paid \$1.50 for a book and marked it to be sold for \$2.00. If he discounted $12\frac{1}{2}$% from his marked price, how much did he gain?

13. An agent sold 165 bales of cotton, each bale weighing 380 lb., at 13¢ a pound, on a commission of $3\frac{1}{4}$%. What was his commission?

1. A man rents a house at the rate of $40 a month. How much *rent* ought he to pay for the use of the house for a year?

2. A borrowed $400 of B at the rate of $6 for each $100 for 1 year. How much *interest* ought A to pay B for the use of his money?

3. **Interest** is money paid for the use of money.

4. The **principal** is the sum of money on which interest is paid.

5. The **amount** is the sum of the principal and interest.

6. Interest at 6% means 6¢ on each dollar for 1 year.

7. When one man borrows money of another he usually gives him a written promise to repay the money. This written promise is called a **promissory note.**

A PROMISSORY NOTE

$ 500. Boston, Sept. 1, 1910

On demand I promise to pay to the order of

George F. Cushman

Five Hundred Dollars

with interest at 6%.

Value received. John C. Taylor.

8. The one who promises to pay is called the **promisor, maker,** or **borrower.**

9. The one to whom the promise is paid is called the **promisee, payee,** or **lender.**

10. The sum of money named in the note is called the **face.**

1. What is the interest on \$600 at 5% for 4 years?

\$600
.05
\$30.00 int. for 1 yr.
4
\$120.00 int. for 4 yr.

Multiply the principal by the rate to find the interest for 1 year. Then multiply the interest for 1 year by the number of years.

ORAL EXERCISES

At 4%, 5%, and 6%, find the interest on:

2. \$800 for 2 yr.
3. \$500 for 3 yr.
4. \$1000 for 4 yr.
5. \$700 for $2\frac{1}{2}$ yr.
6. \$1200 for $1\frac{1}{2}$ yr.
7. \$900 for 2 yr.
8. \$2000 for 3 yr.
9. \$3000 for 1 yr.
10. \$700 for 4 yr.
11. \$400 for $1\frac{1}{2}$ yr.
12. \$1500 for 2 yr.
13. \$200 for $2\frac{1}{2}$ yr.

WRITTEN EXERCISES

Find the interest on:

14. \$247.50 for 5 yr. 6 mo. at 6%.
15. \$813.75 for 8 yr. 3 mo. at 5%.
16. \$1708.20 for 4 yr. 6 mo. at $4\frac{1}{2}$%.
17. \$265.92 for 5 yr. at $3\frac{1}{2}$%.
18. \$562.13 for 3 yr. 6 mo. at 6%.
19. \$6375.56 for 2 yr. 4 mo. at $5\frac{1}{2}$%.
20. \$2000 at $2\frac{1}{4}$ yr. at 5%.
21. \$4120 for $3\frac{3}{4}$ yr. at 6%.
22. \$460.50 for $3\frac{1}{2}$ yr. at $4\frac{1}{2}$%.
23. \$1250 for 2 yr. 3 mo. at $3\frac{1}{2}$%.
24. \$2275 for 2 yr. 8 mo. at 4%.
25. \$1840 for 1 yr. 6 mo. at 5%.
26. \$9750 for 3 yr. 4 mo. at 6%.
27. \$8290 for 2 yr. 3 mo. at $4\frac{1}{2}$%.

There are many methods of computing interest, no one of which is best for all examples. In this book we shall confine ourselves to the method illustrated in Lesson 39, and to the following *60 day method*, also called *banker's method*. For Exact Interest see Lesson 250.

1. Since the interest on \$1 for 12 mo. is \$.06, the interest for 2 mo. or 60 da. will be \$.01. Since the interest on \$1 for 60 days is \$.01, the interest for 6 days will be 1 mill (\$.001).

Learn: To find the interest at 6% on any sum of money for 2 mo. or 60 da., move the decimal point two places to the left. To find the interest for 6 days, move the decimal point three places to the left.

2. Find the interest on \$846 for 4 yr. 5 mo. 12 da. at 6%.

\$8.46 = int. for 2 mo.
.846 = int. for 6 da.
50.76 = int. for 1 yr.
152.28 = int. for 3 yr.
16.92 = int. for 4 mo.
4.23 = int. for 1 mo.
1.692 = int. for 12 da.
\$225.882 = int. for 4 yr. 5 mo. 12 da.

By moving the point find the interest for 2 mo. and also for 6 days. Draw a line to separate these sums from the work below. Take such multiples or parts of these sums as will give the interest for the required time. Do the work mentally, not writing multipliers.

3. Find the interest on \$734 for 115 days at 6%.

\$7.34 = int. for 60 da.
3.67 = int. for 30 da.
1.223 = int. for 10 da.
1.223 = int. for 10 da.
.611 = int. for 5 da.
\$14.067 =
\$14.07

First find the interest for 60 da. The interest for 30 da. will be ½ of the interest for 60 da. The interest for 10 da. will be ⅓ of the interest for 30 da. The interest for 5 da. will be ½ of the interest for 10 da. A little practice will make you skilful in discovering the best way to find the interest for different periods. It is sufficiently accurate to retain only mills in the partial results, and express the final result to the nearest cent.

Find the interest at 6% on:

4. \$5900 for 3 yr. 6 mo. 17 da.

5. \$16,420 for 9 mo. 24 da.

6. \$874.25 for 90 days.

7. \$418.50 for 75 days.

1. Find the interest on \$2184 from Jan. 24, 1910, to April 5, 1913, at 6%.

A. From Jan. 24, '10 to Jan. 24, '13 = 3 yr.
From Jan. 24, '13 to Mar. 24, '13 = 2 mo.
From Mar. 24, '13 to April 5, '13 = 12 da.

B.
$$\begin{array}{rrr} 1913 & -4 & -5 \\ 1910 & -1 & -24 \\ \hline 3\text{ yr.} & -2\text{ mo.} & -11\text{ da.} \end{array}$$

In most business operations 30 days is considered a full month and the time between dates is found as in B. For the purpose of finding interest, method A is preferred as more accurate. From a study of this solution you will see that we first find the full years, then the full months, then the exact days remaining. In March, after the 24th, there are 7 days. 7 da. and 5 da. are 12 da. Do the work mentally, writing only the result.

2. To find the interest at any given per cent, first find the interest at 6%, then add to or subtract from it such parts as will give the required per cent, as, $3\% = \frac{1}{2}$ of 6%; $4\% = 6\% - \frac{1}{3}$ of 6%; $4\frac{1}{2}\% = 6\% - \frac{1}{4}$ of 6%; $5\% = 6\% - \frac{1}{6}$ of 6%; $7\% = 6\% + \frac{1}{6}$ of 6%; $7\frac{1}{2}\% = 6\% + \frac{1}{4}$ of 6%; $8\% = 6\% + \frac{1}{3}$ of 6%.

Find the interest on:

3. \$246.25 from Aug. 10, 1909, to June 8, 1913, at $4\frac{1}{2}\%$.
4. \$408.90 from June 18, 1910, to Oct. 20, 1912, at 6%.
5. \$540.50 from Jan. 8, 1910, to Feb. 2, 1914, at 5%.
6. \$124.84 from Nov. 10, 1909, to Nov. 16, 1913, at 6%.
7. \$264.60 from Feb. 16, 1910, to Aug. 2, 1912, at 4%.
8. \$647.28 from Dec. 15, 1911, to Nov. 5, 1913, at 6%.
9. \$762.40 from June 8, 1911, to Sept. 14, 1912, at $4\frac{1}{2}\%$.
10. \$751.26 from Nov. 2, 1911, to Aug. 17, 1913, at 6%.
11. \$636.20 from Dec. 7, 1909, to Sept. 19, 1911, at 4%.
12. \$345.60 from Nov. 15, 1910, to June 25, 1911, at 6%.
13. \$234.50 from Oct. 7, 1910, to May 2, 1912, at $4\frac{1}{2}\%$.
14. \$123.40 from Sept. 1, 1911, to April 21, 1912, at 6%.
15. \$912.30 from Aug. 11, 1910, to March 2, 1912, at 5%.

1. A man gained $16\frac{2}{3}\%$ by selling his horse for \$105. Find the cost of the horse.

2. If 4 lb. of cheese cost 48¢, how much cheese can you buy for 3 cents?

3. What per cent is lost on goods sold at $\frac{3}{4}$ of their cost? At $\frac{5}{8}$ of their cost? At $\frac{5}{6}$ of their cost? At half price?

4. A man is 72 years old, and $12\frac{1}{2}\%$ of his age is 25% of his son's age. What is the son's age?

5. When 100 shares of R. R. stock are sold for \$17,650, what is the price of one share?

6. If you had \$2 in the morning, but spent during the day 35 cents, a half-dollar, a quarter, and a dime, how much money would you have at night?

7. What day of what month is the 75th day of every year not a leap year? What change does leap year make?

8. What effect is made on an integer by annexing a cipher? By dropping a cipher?

9. What effect is made on a decimal by annexing a cipher? By prefixing a cipher?

10. What change is made in a decimal by moving the point one place to the right? By moving the point one place to the left?

11. The quotient is 45. What would the quotient be if the dividend was multiplied by 2?

12. The quotient is 9. What would the quotient be if the divisor was multiplied by 3?

13. The quotient is 12. What would the quotient be if the divisor was $\frac{1}{6}$ of what it is now?

14. The quotient is 17. What would the quotient be if both dividend and divisor were multiplied by 5?

15. The quotient is 21. What would the quotient be if both divisor and dividend were divided by 7?

16. An article that cost \$40 was sold for \$45. What was the gain per cent?

The questions have been omitted from the problems in this lesson that the pupils may gain power in determining, first, what can be found from given data and, second, *how* to find it. Do not ask the questions for the pupils. Lessons designed for this purpose will be marked simply statements.

1. A mowing machine was sold for \$36 at a loss of 25%.

2. A house was sold for \$2500, at a gain of 25%.

3. A pile of wood is 18 ft. 6 in. long, 4 ft. wide, and 7 ft. 9 in. high. It cost \$8 a cord.

4. Working on a commission of $2\frac{1}{2}\%$, an agent sold 400 bales of cotton, each weighing 480 lb., at 32¢ a pound.

5. A house which cost \$4800 rents for \$24 a month. The expenses on it are \$48 annually.

6. The selling price of an article was \$125.45, and the profit was $8\frac{1}{3}\%$.

7. A dealer sold a horse at $12\frac{1}{2}\%$ loss, and lost \$25.

8. A dealer lost 16% by selling goods for \$4200.

9. A man spent $\frac{8}{21}$ of his money, and invested $\frac{4}{15}$ in business. He had \$1850 left.

10. A flower bed is in the form of a semi-circle. Its radius is 24 ft.

11. A rectangle 150 ft. by 120 ft. has in the center a rectangular grass plot 80 ft. by 60 ft. The rest is covered with gravel 8 in. deep at 5¢ a cubic foot.

12. A bill of goods amounting to \$1000 was sold at discounts of 27% and 10%.

13. After deducting his commission of 5%, an agent returned to his employer \$1436.40.

14. \$468 was on interest at $4\frac{1}{2}\%$ for 5 yr. 9 mo.

15. If the divisor were one third what it is, the quotient would be 948.

16. A bin $8\frac{1}{2}$ ft. by 6 ft. by 5 ft. is $\frac{3}{4}$ full of oats. They are worth 36¢ a bushel.

17. The area of a triangle is 540 sq. ft., and the altitude is 108 ft.

1. If $\frac{1}{4}$ yd. of cloth costs $1\frac{1}{2}$ cents, what will 12 yd. cost?

2. If 3.5 acres cost \$35, what will 40 acres cost?

3. A man sold an automobile for \$180 less than it cost him and lost 20%. What did the automobile cost?

4. A dealer sold a harness for \$48. If this was 20% below cost, find the cost.

5. A boy had 48 cents and spent $\frac{4}{6}$ of it. How many cents had he left?

6. If a horse travels 6 miles an hour, how far will he travel in $4\frac{2}{3}$ hours?

7. A store is insured to the amount of \$5000, at $1\frac{1}{2}$%. What is the premium?

8. A man sold $\frac{1}{4}$ of his farm. Afterwards he bought 13 acres and then had 43 acres. How many acres were in the farm at first?

9. At 7%, find the interest on \$40 for a year.

10. Two men can do a piece of work in 3 days. If one man can do it in 5 days, in how many days can the second man do it?

11. A is 30 ft. ahead of B, but B runs 5 ft. while A runs 2 ft. How many feet will B run to overtake A?

12. A pedler bought potatoes at 60¢ a bushel, and sold them at 25¢ a peck. What was his gain per cent?

13. What is the least common multiple of 8, 12 and 24?

14. A farmer sold his wheat for \$267, and his oats for \$234. How much did he receive for both?

15. If $3\frac{2}{5}$ bbl. of flour cost $\$20\frac{2}{5}$, what will $6\frac{4}{5}$ bbl. cost?

16. What kind of numbers can be subtracted?

17. What is the result of an example in subtraction called?

18. When fractions are unlike, what must be done before subtracting?

19. A merchant sold a chair for \$3.60, which was 150% of its cost. What was the cost?

1. Find the net price of 15 reams note paper at \$1.25 and 25 reams letter paper at \$1.75 with 30% and 12½% off.

2. Change $\frac{1}{16}$ of a mile to rods.

3. A tank 18 ft. long and 15 ft. wide, requires 96 sq. yd. of lead to line the sides and bottom. How deep is it?

4. Multiply seventy-eight ten-thousandths by five hundredths, and divide the product by thirteen thousandths. Reduce the quotient to a common fraction.

5. The fore-wheels of a carriage are 10½ ft. in circumference and the hind-wheels 14 ft. How many more revolutions will one make than the other in going ½ mile?

6. What will it cost to dig a cellar 72 ft. long, 35 ft. wide, and 6 ft. deep, at 62½¢ a cubic yard?

7. Find the interest on \$2862 for 93 days at 6%?

8. Add: .96; 7.3004; 8010; .00093; 1.2465.

9. From 10.101 take 1.0999.

10. A field in the form of a trapezoid contains 11¼ acres. If the parallel sides are 60 rd. and 40 rd., what is the width of the field?

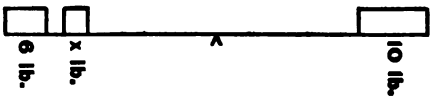
11. A stand-pipe is 30 ft. in diameter and 80 ft. high. How many square feet of sheet iron did it take to make it? How much will it cost to paint the convex surface at 28¢ a square yard?

12. A bin is 10 ft. long, 8 ft. wide, and 6½ ft. deep. If it is ¾ full of wheat, how many bushels of wheat are there?

13. How many board feet are there in 14 beams 15 ft. long, 16 in. wide, and 9 in. thick?

14. A lawn tennis court is 78 ft. long and 27 ft. wide. At each end there is a margin of grass 12 ft. wide, and at each side a margin 6 ft. wide. It cost 50¢ a square yard to turf the margins and 35¢ a square yard to gravel the court. Find the entire cost.

15. A wagon upon which 4-ft. wood was piled was 12 ft. long. How high was the wood if there were 2½ cords?



1. We see that the x weight must weigh 4 lb. in order that the two may balance the 10 lb. weight.

2. This may be written x lb. + 6 lb. = 10 lb. or $x + 6 = 10$. This is called an **equation.** $x + 6$ is the **first member,** and 10 is the **second member.**

3. You learned last year that the equality is not changed when:

a. The same number is added to both members.

b. The same number is subtracted from both members.

c. Both members are multiplied by the same number.

d. Both members are divided by the same number.

a.
$$\begin{aligned} x - 5 &= 8 \\ 5 &= 5 \\ \hline x &= 13 \end{aligned}$$
By adding 5 to both members we get rid of the negative 5 in the first member.

b.
$$\begin{aligned} x + 2 &= 10 \\ 2 &= 2 \\ \hline x &= 8 \end{aligned}$$
In this equation we wish to get rid of the positive 2, so we subtract 2 from both members.

c.
$$\begin{aligned} \tfrac{1}{2}x &= 5 \\ x &= 10 \end{aligned}$$
In this equation we wish to get rid of the fraction, so we multiply both members by 2. $2 \times \frac{1}{2}x = x$. $2 \times 5 = 10$.

d.
$$\begin{aligned} 3x &= 15 \\ x &= 5 \end{aligned}$$
We know the value of $3x$. We wish to know the value of x, so we divide both members by 3.

Find the value of x in the following and state which of the four rules above you used:

4. $x - 3 = 9$	**5.** $x + 5 = 12$	**6.** $x + 4 = 8$
7. $\frac{1}{3}x = 5$	**8.** $7x = 21$	**9.** $\frac{1}{2}x = 6$
10. $x - 7 = 4$	**11.** $x + 5 = 11$	**12.** $x - 2 = 6$

1. Charles and Henry together had 72 cents. If Charles had 3 times as many as Henry, how many did each have?

Let x = No. cents Henry had.
then $3x$ = No. cents Charles had.
and $x + 3x = 72$ cents.
$4x = 72$ cents.
$x = 18$ cents, that Henry had.
$3x = 54$ cents, that Charles had.

Always let x stand for the number that you need to know most in order to find other numbers depending upon it. Study the solution until you understand each step and know why each equation is given.

2. A farmer sold a horse and cow for \$240. He sold the horse for 5 times as much as the cow. How much did he receive for each?

3. Divide 25 into two parts, so that one part shall be $\frac{2}{3}$ of the other part.

(1) Let x = larger part.
(2) then $\frac{2}{3}x$ = smaller part.
(3) then $x + \frac{2}{3}x = 25$
(4) $3x + 2x = 75$
(5) $5x = 75$
(6) $x = 15$. $\frac{2}{3}x = 10$.

If x is one part and $\frac{2}{3}x$ is the other part, then the 2 parts together will equal the number 25. What rule do you apply to change the third equation to the fourth?

4. Three-sevenths of a certain number exceeds $\frac{1}{4}$ of the same number by 25. What is the number?

Let x = the number.
then $\frac{3}{7}x - \frac{x}{4} = 25$
$12x - 7x = 700$
$5x = 700$
$x = 140$

In this example, to get rid of the fractions we must multiply both members by 28, the least common multiple of the denominators.

5. After a boy had spent $\frac{1}{4}$ and $\frac{1}{5}$ of his money he had \$1.32. How much money had he at first?

6. A horse was sold for \$96 at a gain of 20%. Find the cost of the horse.

Let x = cost of the horse.
$\frac{20}{100}x$ or $\frac{x}{5}$ = the gain.
then $x + \frac{x}{5} = \$96$.

1. A general placed 4800 men in 3 regiments so that the 2d regiment had twice as many as the 1st regiment and the 3d regiment had as many as both the others. How many men were placed in each regiment?

2. If you add 14 to 3 times a certain number, you will obtain 32. What is the number?

3. A, B, and C, earned $8580. A earned twice as much as B and B earned four times as much as C. How much did each earn?

4. What number increased by $\frac{1}{2}$ of itself will equal $37\frac{1}{2}$?

5. What number increased by $\frac{1}{2}$ and $\frac{1}{6}$ of itself will equal 34?

6. What number increased by $\frac{1}{3}$ and $\frac{1}{4}$ of itself will equal 38?

7. What number diminished by $\frac{2}{5}$ and $\frac{3}{7}$ of itself equals 210?

8. What number must be added to $\frac{2}{3}$ of itself that the sum may be 55?

9. Esther is $\frac{1}{3}$ as old as Jessie, and the sum of their ages is 20 years. What is the age of each?

10. The sum of two numbers is 16 and their difference is 2. What are the numbers?

11. Thomas has 9 more marbles than John and together they have 35 marbles. How many marbles has each?

12. What number must be added 3 times to itself that the sum may be 160?

13. A boy lost $\frac{4}{5}$ of his marbles and had 35 marbles left. How many had he at first?

14. If $\frac{1}{2}$ of a number is added to $\frac{1}{4}$ of the same number, the sum will be 36. What is the number?

15. Divide 27 into three parts, so that the first shall be 5 greater and the second 5 less than the third.

16. The perimeter of a rectangle is 42 inches. If it is twice as long as it is wide, how long is it?

1. Three men, A, B, and C, formed a company with a capital of $8000. B put in 4 times as much as A, and C 3 times as much as A. How many dollars did each put in?

2. The sum of 2 numbers is 99, and the greater is twice the less. What are the numbers?

3. Divide the number 88 into three parts, so that the second shall be 3 times the first, and the third 4 times the first.

4. Four men have together $480. B has 4 times as much as A; C has 6 times as much as B, and D has $\frac{1}{8}$ as much as C. How many dollars has each?

5. A certain number added to 7 times itself equals 64. What is the number?

6. A company of 300 persons consists of men, women, and children. There are 2 times as many children as men, and 3 times as many women as men. How many women are there?

7. A man bought an equal number of lemons and oranges for $6.25. For the lemons he paid 2¢ each, and for the oranges 3¢ each. How many of each did he buy?

8. The larger of 2 numbers is 5 more than the smaller, and their sum is 17. What are the numbers?

9. Divide 20 in two parts so that the larger part may be 4 more than the smaller part.

10. Anna is one-half as old as Mary, and the sum of their ages is 21. What is the age of each?

11. The difference between two numbers is 7, and their sum is 23. What are the numbers?

12. Three men are in business together. Divide their gain of $9000 in the proportion of 3, 5, and 7.

13. Two men bought an automobile for $2400. If one man paid $3 to the other man's $5, how much did each pay?

14. Separate 153 into parts proportional to 8 and 9.

1. The product of a and b is written ab. The product of 5, x, and y is written $5xy$.

2. Write the product of 8 and c. Of 4, a, and d. Of 7, x, y, and z.

3. The product of a and a is not usually written aa, but a^2, and is read a squared or the square of a.

4. When $a = 4$, and $b = 2$, find the value of $3ab$.
This means $3 \times 4 \times 2 = 24$.
When $a = 4$, $b = 3$, $c = 1$, $x = 2$, $y = 5$, $z = 6$, find the value of:

5. $4a$	6. $3c$	7. $2y^2$	8. a^3	9. $2b^3$	10. $5x^4$
11. $7z$	12. c^2	13. $2z^2$	14. x^3	15. $4by$	16. $7x$
17. $2ab$	18. $3bc$	19. $4cx$	20. $5xy$	21. $6yz$	22. $2ax$

23. Express 4 increased by 5; a increased by b.

24. Express 9 diminished by 5; a diminished by b.

25. How much does b lack of being 15?

26. If a man walks 3 miles an hour, how many miles will he walk in x hours?

27. If a man walks x miles an hour, how many miles will he walk in c hours?

28. If one part of 12 is 7, what is the other part?

29. If one part of 15 is x, what is the other part?

30. If a pear costs $2x$ cents and a peach $3x$ cents, what will represent the cost of both?

31. If $3x = 9$, what does $3x + 5$ equal?

32. If $x - 4 = 8$, what does x equal?

33. If $x = 2$, what will $3x$ equal?

34. If $x = 7$, what will $x - 3$ equal?

35. If $x + 4 = 6$, what does x equal?

36. If $\frac{1}{5}x = 20$, what does $4x$ equal? What does x equal?

37. Let c represent the circumference of a cylinder and a its altitude. Find its convex surface.

1. The greater of 2 numbers is twice the less, and the sum of the numbers is 135. What are the numbers?

2. The sum of the ages of a mother and daughter is 36 years, and the age of the mother is 8 times that of the daughter. What is the age of each?

3. Divide 36 into three parts so that the first part shall be 3 times the second, and the second 2 times the third.

4. John is 2 times as old as Henry and the sum of their ages is 18 years. What is the age of each?

5. Alice had a certain sum of money, and earned twice as much. After spending 9 cents, she had 21 cents left. How much money did she earn?

6. The sum of two numbers is 155, and the greater is 4 times the less. What are the numbers?

7. The difference between 2 numbers is 132, and the greater is 4 times the less. What are the numbers?

8. One number is 4 times another, and their difference is 30. Find the numbers.

9. A man sold 3 houses of equal value and a barn for $24,000. If the barn brought $1600 less than a house, what was the price of each?

10. The sum of two numbers is 58, and the greater is 7 less than 4 times the smaller. What are the numbers?

11. Divide the number 133 into two parts such that one part is 15 more than the other part.

12. If 38 is added to 3 times a number, the sum will be 98. What is the number?

13. A farmer bought a horse, a cow, and a calf for $200. The cow cost six times as much as the calf, and the horse three times as much as the cow. What was the cost of each?

14. Samuel has a certain number of marbles, and John has 15 more than Samuel. If together they have 103 marbles, how many has each?

When $a = 4$, $b = 1$, $c = 3$, $x = 5$, $y = 7$, $z = 0$, find the value of:

1. $3x + 5z$	2. $3y - 4z$	3. $3a - 5b$	4. $5b + c$
5. $5c + 3a$	6. $4x - 2y$	7. $2y - 3b$	8. $2x + 5a$
9. $3y - 5a$	10. $x^2 - 3a$	11. $b^3 + a^2$	12. $xz - 2b$

13. If the length of a rectangle is x in. and the width y in., what is the perimeter of the rectangle? What is the difference between the length and breadth?

14. Express the area of this same rectangle.

15. Express the square of x; the cube of y; the fourth power of a.

16. x and y represent two numbers of which x is the greater. What will represent their sum? Their difference? Their product? Their quotient?

17. A triangle has a base a feet long and an altitude $2b$ feet long. What is its area?

18. The diameter of a circle is $2a$ feet and its circumference $2b$ feet. What is its area?

19. A boy had a kite-string x feet long. From it he cut 3 pieces, each y feet long. How many feet did he have left?

20. Write as an equation the statement: The excess of 17 over 8 is 9.

21. Write as an equation the statement: The excess of x over y is z.

22. $3x + 2 = 5$. What does x equal?

Indicate by use of exponents:

23. The fifth power of b.

24. The cube of x plus the square of y.

25. The number of quarts in 4 gallons.

26. The number of inches in 12 feet.

27. The number of seconds in a degree.

1. Three men together have \$68,400. If the second has \$3000 more than the first and the third \$2400 more than the second, how much has each?

2. What number must be added 3 times to itself that the sum may be 40?

3. George has 7 marbles more than William and they both have 29 marbles. How many has each?

4. Three men have \$63. A has \$3 more than C, and C has twice as many dollars as B. How many dollars has each?

5. A number and one third of the same number added together are 32. What is the number?

6. The difference between the ages of a father and his son is 24 years, and the father's age is three times that of the son. What is the age of each?

7. A farmer has 3 flocks of sheep, numbering in all 320 sheep. The flocks are in relative size as the numbers 1, 2, and 5. How many sheep are there in each flock?

8. A, B, and C entered into partnership, and their profits were \$1,612.50. If their capital invested is proportional to 16, 9, and 5, how should the gain be divided?

9. Three contractors agreed to dig a canal for \$1010. A furnished 60 men, B 200 men, and C 144 men. Of the sum received, how much should A, B, and C each receive?

10. Divide \$950 among three persons, giving the first \$3 as often as you give the second \$9, and the third \$13.

11. Divide \$1071 among A, B, and C, so that their shares shall be in the ratio of 5, 7, and 9.

12. Three men buy a mill for \$105,750. The sums they pay are to each other as 3, 5, and 7. What sum does each pay?

13. A horse and carriage are valued at \$420. If $1\frac{1}{3}$ times the value of the horse is equal to the value of the carriage, find the value of each.

1. Compare 9 with 3. Compare $\frac{1}{2}$ with $\frac{1}{4}$.

2. The comparison of one object or quantity with another object or quantity of the same kind is called **ratio.**

3. Ratio is found by dividing the first number by the second, and is expressed in three ways:

a. By the sign of division, as $9 \div 3$.
b. By the colon, as 9:3, read 9 is to 3.
c. By a fraction, as, $\frac{9}{3}$.

4. The first term of a ratio is called the **antecedent** and the second term is called the **consequent.**

5. What is the ratio of 18 to 9? 35 to 7? 16 to 4? 5 to 15? 9 to 12? 16 to 24?

6. 48 is the antecedent and 8 is the consequent. What is the ratio?

7. 12 is the consequent and 5 is the ratio. What is the antecedent?

8. 36 is the antecedent and 3 is the ratio. What is the consequent?

9. What kind of numbers can be compared?

10. When is the ratio a fraction? When is it an integer?

11. What is the ratio of .27 to .3? .12 to .2? .5 to .25?

12. Divide 64¢ between 2 girls in the ratio of 3 to 5.

$\frac{3}{8}$ of $64¢ = 24¢$
$\frac{5}{8}$ of $64¢ = 40¢$

Each time you give one girl 3¢, you must give the other 5¢. Hence the first girl has 3¢ out of every 8¢, or $\frac{3}{8}$ of the money, and the second girl must have $\frac{5}{8}$ of it.

13. Divide \$1.21 between 2 boys in the ratio of 5 to 6.

14. Divide 45 marbles between 2 boys in the ratio of 7 to 8.

15. There are 45 children in room 9, and the number of boys is to the number of girls as 5 is to 4. How many girls are there?

16. If $\frac{1}{2}$ in. on a map represents a distance of 50 miles, how many miles will $1\frac{1}{2}$ in. represent?

1. What is the ratio of 9 to 3? What is the ratio of 12:4?

2. These two ratios being equal may be written:

$9:3 = 12:4$. It may be read in 2 ways: The ratio of 9 to 3 equals the ratio of 12 to 4, or, 9 is to 3 as 12 is to 4.

3. **Proportion** is an equality of ratios.

4. In the proportion $9:3 = 12:4$, name the antecedent of each ratio. Name the consequent of each ratio.

5. In every proportion the first and fourth terms are called the **extremes**. The second and third terms are called the **means**.

6. $9:3 = 12:4$. Name the extremes and means in this proportion. Multiply the extremes together. Multiply the means together. Compare the two products.

Learn: In every proportion the product of the means equals the product of the extremes.

7. Either extreme equals the product of the means divided by the other extreme, as $8:4 = 6:x$. $8x = 24$. $x = 3$.

8. Either mean equals the product of the extremes divided by the other mean, as, $9:3 = x:4$. $3x = 36$. $x = 12$.

Find the missing term in each proportion:

9. $x:2 = 8:4$ 10. $10:2 = 15:x$

11. $14:x = 10:5$ 12. $15:5 = 12:x$

13. If 3 tons of coal cost \$16, how much will 6 tons cost?

$3 \text{ tons} : 6 \text{ tons} = \$16 : \$x$

$3x = \$96$

$x = \$32$

A. Let x = the cost of 6 tons and write the ratio $\$16 : \$x$. Since the quantity of coal is directly proportional to the money expended, we write our first ratio 3 tons (that cost \$16):6 tons (that cost $\$x$).

B. Put $\$x$ in the fourth place. Since we are seeking for cost we put \$16 in the third place, because the third and fourth terms form a ratio and must be of the same kind. We know that 3 tons cost \$16 and that 6 tons will cost *more* dollars. This shows us that the consequent of our second ratio will be larger than its antecedent, therefore to have an equality of ratios we must make the consequent of our first ratio larger than its antecedent.

1. If 5 men can do a piece of work in 13 days, in how many days can 65 men do it?

A. Analysis.

5×13 days $= 65$ days.

$\frac{1}{65}$ of 65 days $= 1$ day.

Since 5 men can do the work in 13 days, 1 man can do it in 5×13 days or 65 days. Since 1 man can do the work in 65 days, 65 men can do it it in $\frac{1}{65}$ of 65 days or 1 day.

B. Proportion.

$65 \text{ men} : 5 \text{ men} = 13 \text{ d.} : x\text{d.}$

$$\frac{5 \times 13}{65 \times x} = 1 \text{ day.}$$

In arranging your proportion use either method given in the preceding lesson. This is an example of *inverse* proportion. In solving use cancelation.

Solve the problems in this lesson by analysis and by proportion:

2. If a quantity of provisions will last 630 men in a garrison 108 days, how long will the same quantity last 210 men?

3. If $1\frac{1}{5}$ yd. cost $\$1\frac{3}{4}$, what will 81 yd. cost?

4. If $6\frac{1}{2}$ cd. of wood cost \$28.60, how much can be bought for \$514.80.

5. If 24 lb. of butter can be made from 360 qt. of milk, how much milk will be required to produce 100 lb. of butter?

6. If $5\frac{1}{2}$ bu. of wheat cost \$4.95, how many bushels can be bought for $\$121\frac{1}{2}$?

7. If 20 men earn \$1280 in 8 days, how much can 36 men earn in the same time? How much can they earn in 24 days? In problems like this use your answer for the first part, in solving the second part.

8. If $8\frac{1}{4}$ tons of coal cost $\$37\frac{1}{8}$, what will $27\frac{1}{2}$ tons cost?

9. The wages for 20 men for 12 weeks are \$870. What are the wages of 8 men for the same time? Find the wages of 8 men for 4 weeks.

10. If 7 men can build $6\frac{3}{4}$ rd. of wall in $15\frac{1}{2}$ days, in how many days can 14 men build the same wall? How many rods could 14 men build in $15\frac{1}{2}$ days?

11. If $5\frac{7}{8}$ yd. of cloth cost \$4.70, what will $17\frac{5}{8}$ yd. cost?

1. Divide 36¢ into parts proportional to 3, 4, and 5.

Let $3x$, $4x$, and $5x$ = the parts.
Then $3x + 4x + 5x = 36¢$.
$12x = 36¢$ $x = 3¢$.
$3x = 9¢$ $4x = 12¢$ $5x = 15¢$.

2. Two men gained \$1100 in business. If they divided this in the proportion of 4 to 7, find the share of each.

3. Two men received \$65 for a piece of work. If one man worked 7 days and the other 6 days, how should the money be divided between the two men?

4. Divide \$4680 among three partners so that their shares shall be in the proportion of 2, 3, and 4.

5. An association of two or more men in business is called a **partnership**. The association is called a **firm** or **company**. The persons so associated are called **partners**.

6. Since gains and losses of a firm are shared in proportion to the amount of capital invested by each partner, partnership is only one form of proportion.

7. Three men purchased a store paying as follows: A, \$2000; B, \$4000; C, \$3000. Divide the gain of \$1800 between the three men.

\$2000 + \$4000 + \$3000 = \$9000, the whole capital.
A's share of the capital is $\frac{2000}{9000} = \frac{2}{9}$
Hence A's share of the gain must be $\frac{2}{9}$ of \$1800 = \$400. Find B's and C's shares in the same way.

8. Three men bought a house for \$5000. A paid \$1000, B, \$2400, and C, \$1600. If they rent it for \$600 a year, find each one's share of the rent.

9. Two men rent a farm for \$20.40. The first pastured 32 cattle and the second 36 cattle. How much should each man pay?

10. Three boys hire a bicycle for \$48. The first boy uses it 4 weeks, the second 5 weeks, and the third 3 weeks. How much should each boy pay?

1. Find 50% of 600 yd. $8\frac{1}{3}\%$ of 72 bu. 40% of 80 tons. 5% of \$40.

2. What are the terms of a ratio called?

3. What is the area of a rectangle whose base is 15 ft. and altitude 12 ft?

4. If 10 tons of coal cost \$55, what will 12 tons cost?

5. The goods in a store are worth \$3000, and are insured for $\frac{2}{3}$ of their value at $\frac{1}{2}\%$ premium. What will be the owner's loss if the goods burn? What will be the loss of the insurance company?

6. If $\frac{1}{2}$ of a number is added to itself the sum will be 30. What is the number?

7. If $\frac{1}{3}$ of a number is added to itself the sum will be 24. What is the number?

8. A man who has failed in trade is able to pay only 62¢ on a dollar. If he owes your father \$100, how much will your father lose?

9. If 3 men earn \$18 in a week, how many dollars will 4 men earn in a week? How many dollars will 4 men earn in 5 weeks?

10. Two men are 124 miles apart and traveling toward each other. If one travels 28 miles a day and the other 34 miles a day, in how many days will they be together?

11. If the two men in example 10 are traveling in the same direction, in how many days will they be together?

12. If 3 oranges cost 6 cents, what will a dozen oranges cost?

13. Three times 10 and $\frac{3}{5}$ of 10 are how many?

14. State example 13 in another way.

15. If 3 men can do a piece of work in $7\frac{7}{9}$ days, how long will it take 7 men to do it?

16. $5\frac{3}{7}$ is $\frac{1}{3}$ of what number? $8\frac{5}{9}$ is $\frac{1}{5}$ of what number?

17. At 20¢ a dozen, what will 18 eggs cost? What will 30 eggs cost?

1. A tree 45 ft. high was broken at such a point that the part broken off was 4 times the length of the part left standing.

2. Goods marked \$64 were sold at $6\frac{1}{4}\%$ discount and 5% off for cash.

3. A commission merchant sold some goods for \$5728 and sent \$5656.40 to his principal.

4. A horse and buggy are worth \$300. The horse is worth 4 times as much as the buggy.

5. If the divisor were $\frac{1}{3}$ what it is, the quotient would be 948.

6. A merchant sold some cloth at \$1.19 a yard and lost 15%.

7. The merchant in example 6 marked the cloth so as to gain 15%.

8. An agent sold goods on 2% commission and remitted to his employer \$4777.50.

9. A man sold an automobile for \$931, thereby losing 5%.

10. The ceiling of a room 15 ft. 6 in. long, and 12 ft. 6 in. wide was painted at 16¢ a square foot.

11. A park 9 rd. square has a walk 9 ft. wide round it.

12. A man bought 20 sheep for \$250 and 20 cows for \$1050. He sold the sheep at a gain of $12\frac{1}{2}\%$ and the cows at a gain of $7\frac{1}{2}\%$.

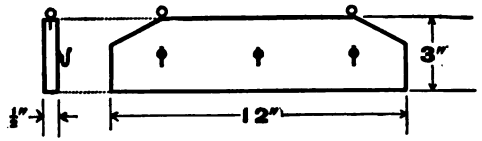
13. A man insured his house for $\frac{3}{4}$ of its value at $\frac{2}{3}\%$. The premium was \$24.

14. A number increased by 22% of itself is 23,546.

15. A pile of wood contains 12 cords. It is 32 ft. long and 4 ft. wide.

16. The product of three numbers is 3135. One of the numbers is $28\frac{1}{2}$, and another is 22.

17. By selling butter at 30¢ a pound on 4% commission, an agent received \$72.

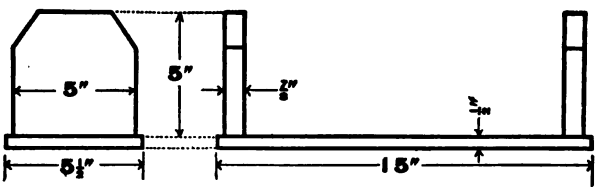


1. What part of a board foot does it take to make the keyboard shown in the drawing?

2. How many board feet will furnish sufficient material for a class of 24 boys?

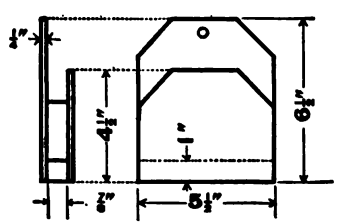
3. The keyboards are made from white-wood which costs 10¢ a board foot. Find the cost of lumber required for the class.

4. Allowing $2\frac{1}{2}$ cents for the hooks and screw rings used on each board, find the cost of each keyboard when completed.

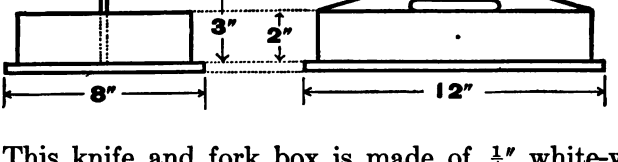


5. From the drawing find each dimension. Add one inch to each linear dimension as an allowance for waste in cutting, and, assuming that all pieces will be cut from stock 6" wide, find how many board feet of lumber will be required for a class of 24 boys.

6. If this book rack is made from California Bay-wood, which costs $300 per thousand feet, find the cost of lumber for each book rack. Find the cost for the class.



7. From the drawing find each dimension. Add one-half inch to each linear dimension and find the amount of lumber needed for 24 boys each to make a letter holder.



1. This knife and fork box is made of $\frac{1}{4}''$ white-wood. Find the dimensions and the area of the bottom board.

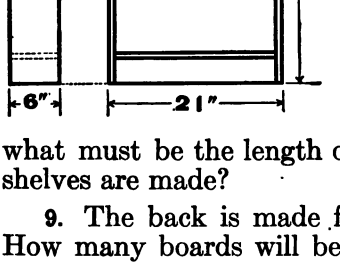
2. The bottom board projects $\frac{1}{2}''$ beyond the sides all the way round. Find the length of each side.

3. The sides are nailed to the ends. Find the length of the ends.

4. In cutting up the lumber $\frac{1}{2}''$ is added to the length of each piece to allow for squaring. How long a piece will be required to make the sides and ends?

5. The handle is cut from a piece $\frac{1}{4}'' \times 3'' \times 12''$. Find how many board feet will be needed to allow each boy of class of 24 to make the box.

6. If the lumber costs \$40 per M, find the cost of the material for the class.

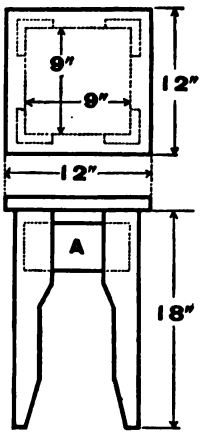


7. Add 1″ to the length of each piece for squaring. Find the dimensions of the board from which the end piece can be cut.

8. The cabinet is made of $\frac{1}{2}''$ white-wood. Find the length of each shelf when finished. Making allowance as in example 7, what must be the length of the piece from which the three shelves are made?

9. The back is made from matched boards 3 in. wide. How many boards will be needed?

10. Find the number of board feet required for one of the cabinets. For 24 cabinets.



1. The list of the number and sizes of pieces required in making an article is called the bill of lumber.

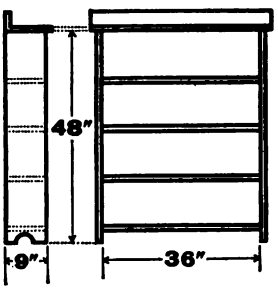
2. The legs of this tabouret are made of two pieces $\frac{1}{2}'' \times 3''$ nailed together. "A" is the rim made of $\frac{1}{2}'' \times 4''$ pieces fastened within the legs. Notice that there is a sub-top. Find all the other measurements from the drawing.

3. Make out a bill of lumber for this tabouret.

4. Find the cost at 10¢ a board foot of the lumber used in making 21 of these tabourets.

5. What would be the additional cost if the top were 15″ square and the sub-top 12″ square, and the rim correspondingly larger?

6. The top of this bookcase is 9″ wide and projects over each side $2\frac{1}{8}''$. The upright strip at the back of the top is made of a piece $\frac{7}{8}'' \times 3''$ of the same length as the top board. Find out the amount of lumber needed for the top and the 4 shelves, each 8 in. wide.



7. The back is made of $\frac{1}{2}''$ matched boards. Find the lumber for the back, adding $\frac{1}{6}$ to the amount because of the loss in matching.

8. Find the lumber needed for the two sides.

9. Make out a bill of lumber for the entire bookcase.

10. At 12¢ a board foot, find the cost of the lumber for 20 cases.

1. A manual training room in a certain school measures $40' \times 30'$. There are 21 benches, each $4' \times 2'$. What per cent of the room is occupied by the benches?

2. Across one end of the room is a case 8′ high and 3′ deep, divided into 8 compartments. Find the number of cubic feet of space taken up by the case. How many cubic feet are there in each compartment?

3. What per cent of the floor space is covered by the case?

4. On one side of the room are 4 demonstration settees, each occupying about 18 sq. ft. of space. What per cent of the floor space is covered by the demonstration settees?

5. What per cent of the floor space is covered by the benches, case, and settees?

6. The cost of each of the 21 benches was \$9.35, and the cost of the tools for the entire equipment was 80% of the cost of the benches. Find the entire cost.

7. Each bench has the same number and kind of tools, costing \$5.65 a set. How much was invested in extra tools for general use?

8. If the 280 boys, who are under instruction, use \$392 worth of lumber during the year, what is the average cost for each boy for a lesson, if the classes meet 35 times during the year?

9. Besides the lumber there is an extra expense for sandpaper, nails, screws, and sharpening tools. Sandpaper costs 1¢ a sheet, and each sheet is divided into 8 pieces. If each boy uses 2 pieces for each lesson, find the cost of sandpaper for the year.

10. If the cost of screws and nails is \$24.50 for the year, how much is the average cost for each boy for a lesson increased by the cost of the sandpaper, screws, and nails?

The following tables, taken from the U. S. Department of Agriculture, give a menu for one day for a family of 4 adults.

BREAKFAST

	Lb.	Oz.	Cost
Corn meal in mush and cake	0	5	1¢
Milk	0	6	1¢
Sugar	0	2	½¢
Toast	0	10	2½¢
Butter	0	2	3¢

DINNER

	Lb.	Oz.	Cost
Beef roll	3	0	15¢
Potatoes	1	8	2¢
Beets	0	8	1¢
Bread	0	10	2½¢
Butter	0	2	3¢

SUPPER

	Lb.	Oz.	Cost
Baked beans	2	10	6¢
Pork	0	12	6¢
Potatoes (fried)	1	8	2¢
Lard	0	2	1¢
Bread	0	10	2½¢
Butter	0	2	3¢

1. Find the total cost for three meals for the day.

2. Find the total cost for each man for the day.

3. At that rate per day what would the meals cost for the family for a week?

4. Find the total weight of the food consumed.

5. Find the average weight per man of the food consumed.

6. From the cost and weight of butter given, find the price per pound.

7. A bushel of potatoes weighs 60 lb. What part of a bushel was consumed? Find the estimated price of a bushel.

8. A certain family added to the above menu as follows: At breakfast broiled sirloin, 11¢; at dinner tomato soup, 6¢, apple dumpling, 8¢; at supper oranges, 7¢. Find the total addition to the above cost.

1. A family had an annual salary of $2500. They spent 30% for food, 15% for rent, 3% for fuel, 10% for wages, 10% for clothing, 5% for books and travel, 5% for charity and insurance. If they saved the rest how much did they save? Find the sum spent for food, rent, etc.

2. The next year the same family wished to have more money for books and travel and yet save the same amount. This year they spent 25% of their salary for food, 15% for rent, 3% for fuel, 10% for clothing, and 5% for charity and insurance. How much more money did they save for books and travel?

3. Find the cost of furnishing a bedroom, if the following articles were bought: Rug, $4.50; bed and spring, $6.50; mattress, $4.75; 2 pillows @ $2.50; 4 sheets @ $.75; 3 pairs of pillow cases @ $.50; 1 pair blankets, $4.00; 1 spread, $1.25; 1 bureau, $8.50; 2 chairs @ $2.00.

4. Find what per cent the cost would have been increased if the following articles had been purchased: 1 rocker, $3.75; 1 table, $6.00; 1 washstand, $4.75; 1 toilet set, $3.25; $\frac{1}{2}$ doz. towels, $1.60.

5. The following articles were bought for the dining-room: 1 rug, $8.00; 1 table, $13.50; 6 chairs @ $3.00; 1 side board, $12.19; 1 serving-table, $5.75; $2\frac{1}{2}$ yd. table linen, @ $1 a yard; $2\frac{1}{2}$ yd. table linen @ $1.20; 1 doz. napkins, $1.50; 1 doz. nakpins, $2.00; 2 tray cloths @ $.75 each. The cost of furnishing the dining-room was what per cent of the cost of furnishing the bedroom?

6. The dining-room table is 42 in. square. There are 3 extra leaves, each 10 in. wide. Give the dimensions of the table when the three leaves are used.

7. What must be the width and length of the table cloth to allow it to hang down 9 in. on each side?

8. The sheets had a 3-in. hem at the top and bottom and were $2\frac{3}{4}$ yd. long when finished. How many sheets could be made from $17\frac{1}{2}$ yd. of sheeting?

1. Guess at the length of the room. Measure the length. How great a mistake did you make.

2. Walk the length of the room counting your steps. Find the length of your steps.

3. Estimating distances by counting the number of steps is called **pacing.** A pace is usually 3 feet. How much less than a pace was the length of your step?

4. Pace the width of the room. How many feet wide did you make it? Measure the width. How near was your estimate to the correct distance?

5. Make two marks on the floor 1 rod apart. Pace the distance until you can correctly estimate a rod, 2 rods, 3 rods, etc.

6. Cut a string 165 ft. long. How many rods long is it? Call this your measuring string.

7. Estimate by pacing the distance from one street to another. With your measuring string measure the distance, correcting your error if any.

8. Using your measuring string find some object 20 rd. from the schoolhouse. What part of a mile is it?

9. In the same way find an object that is 40 rd. distant. Find one that is 80 rd. distant. One that is 160 rd. distant. In each case tell what part of a mile it is.

10. Since the square rod is a common unit of measure, mark off on the floor a square rod. Study it closely till you have formed a mental picture of its size. This is called visualizing it.

11. Without measuring mark off in the yard a square rod. Measure it to correct your error.

12. In some park or vacant lot mark off a rectangle 16 rd. $\times$ 10 rd. or 20 rd. $\times$ 8 rd. This is 1 acre.

13. Without measuring estimate how much more or less than an acre of land there is in the school yard. Measure and see if you made any error.

TABLE OF U. S. MONEY.

10 mills = 1 cent (¢)
10 cents = 1 dime
10 dimes = 1 dollar ($)
10 dollars = 1 eagle (E)

The standard of value in the U. S. is the dollar. The mill is not coined, but is expressed as a tenth of a cent. The coins of the U. S. are made of bronze, nickel, silver, and gold.

TABLE OF ENGLISH MONEY

4 farthings (far.) = 1 penny (*d*)
12 pence = 1 shilling (*s*.)
20 shillings = 1 pound (£)

The standard of value in England is the pound or sovereign. The farthing is not coined but is expressed as a fraction of a penny. The coins of England are made of copper, silver, and gold. The names and values of coins in Canada are the same as in the United States.

A shilling is equal to $.24 and a pound to $4.8665. In estimating equivalents we call the shilling 25¢ and the pound $5.

1. How many cents are there in 25 dollars? In 25 eagles?

2. How many pence are there in 25 shillings? In 25 pounds?

3. Change 144 pence to shillings. Change 80 shillings to pounds.

4. How many shillings are there in ½ sovereign?

5. Change £1 3*s*. to pence.

6. About what is the value of $100 in pounds sterling? What is its exact value in pounds and hundredths of a pound?

7. How much U. S. money will it take to pay a debt in London of £40 8*s*.?

Change £8 12*s*. 8*d*. to United States money.

12) 8.
20)12.666+
£8.6333+

8*d*. = .666+ shillings. 12.666*s*. = .6333+ pounds. Hence £8 12*s*. 8*d*. equals £8.6333+. Since 1 pound equals $4.8665, 8.633 pounds must equal 8.6333 × $4.8665.

8.6333 × $4.8665 = $——.

Change to United States money:

8. £7 15*s*.
9. £15 10*s*. 6*d*.
10. £12 4*s*. 8*d*.
11. £20 10*s*.
12. £12 15*s*. 3*d*.
13. £75 5*s*. 1*d*.

1. Change \$115.50 to English money.

\$115.50 ÷ \$4.8665 = 23.733 +, the number of pounds.
.733 × 20*s*. = 14.66*s*. .66 × 12*d*. = 7.92*d*.
Hence \$115.50 = £23 14*s*. 8*d*.

2. Give the approximate equivalent in pounds of \$500.

Reduce to English money to the nearest penny:

3. \$7.25 4. \$36.15 5. \$120.10 6. \$2000

The standard of value in Germany is the *mark* which is equivalent in value to \$.2385, approximately \$.25.

7. Change 1000 marks to its exact equivalent in U. S. money. To its approximate equivalent.

8. A traveler carried \$238.50 to Berlin. When changed to marks, how many would he have?

9. A German laborer received 25 *pfennigs* an hour, and worked 9 hours a day for 30 days. His wages for the whole time is equivalent to how much U. S. money? 100 *pfennigs* equal a mark.

The standard of value in France is the *franc*, which is equivalent in value to \$.193, approximately \$.20. In Italy the standard of value is the *lira*, of the same weight and value as the franc.

10. Find to the nearest cent the value of 500 lira. Of 1500 francs.

11. Merchandise invoiced at 120 lira was imported from Italy. Find to the nearest cent its value in U. S. money.

12. The toll on vessels passing through the Suez Canal is $8\frac{1}{2}$ francs a ton. How many francs must be paid by a vessel carrying $3956\frac{1}{2}$ tons? Find its equivalent in U. S. money.

13. One dozen cloaks invoiced at 600 francs was received from Paris and 300 yd. of carpeting, invoiced at 6*s*. a yard, was received from London. Find to the nearest cent the value of all in U. S. money.

1. Find the joint effect of the following 3 forces: A, pulling forward $76\frac{1}{4}$ lb.; B, pulling forward $82\frac{1}{2}$ lb.; and C, pulling backward $84\frac{7}{8}$ lb. To what single force are the three forces equal?

2. A cubic foot of brass weighs 523.8 lb. Find its specific gravity by dividing by $62\frac{1}{2}$ lb. the weight of a cubic foot of water.

3. In £28 15*s*. there are how many shillings? Find the value in U. S. money.

4. How many tons are there in 14,200 lb.?

5. A platform was built to sustain a weight of 45 tons. On the platform is a pile of oak boards 20 ft. long, 10 ft. wide, and 8 ft. high. If the average weight is 54 lb. to each cubic foot, how much more weight could the platform sustain?

6. To what single rate of discount is a discount of 20% and 5% equivalent?

7. A rectangular wheat field is 40 rd. $\times$ 20 rd. The farmer reaps a swath $8\frac{1}{4}$ ft. wide round the field. How many square rods does he reap?

8. If he again reaps a swath $8\frac{1}{4}$ ft. wide round the field, how many rods does he reap the second time round? The third time round? The fourth time round?

9. Make out a receipted bill for the following items:

May 1, 1910, Mr. J. R. Jones bought of Smith Bros., 14 yd. of silk, @ \$1.75; May 10, 58 yd. cotton @ 9¢; May 12, 10 yd. suiting @ \$2.25; May 18, 24 yd. carpet @ $87\frac{1}{2}$¢; June 3, 40 yd. matting @ 35¢; June 7, 5 pairs curtains @ \$5.75.

10. Define antecedent, consequent, proportion.

11. The antecedent is 150, and the consequent $37\frac{1}{2}$. What is the ratio?

12. If a man travels 48 mi. in 12 hr., how many miles can he travel in 60 hr. at the same rate?

1. What is the area of a rectangle 40 ft. × 20 ft?

2. A lot measures 60 ft. by 120 ft. How many posts, 10 ft. apart, will be required for the fence for this lot?

3. How many strips of carpet 1 yard wide are needed for a floor 30 ft. × 18 ft., if the breadths run lengthwise?

4. How many yards long will each strip be? How many yards of carpet will be needed for the room?

5. How many acres are there in a field 80 rd. by 40 rd.?

6. What is a parallelogram?

7. Find the area of a parallelogram whose base is 40 ft. and whose altitude is 20 ft.

8. Define a triangle. Give the rule for finding its area.

9. Find the area of a triangle whose base is 4 ft. and altitude 4 ft.

10. The area of a triangle is 40 sq. ft. If the base is 10 ft., what is the altitude?

11. Define a trapezoid. Give the rule for finding its area.

12. The two parallel sides of a trapezoid are 20 ft. and 10 ft. If the altitude is 8 ft., what is the area?

13. The area of a trapezoid is 60 sq. ft. If the two parallel sides are 12 ft. and 8 ft., what is the altitude?

14. How can you find the diameter of a tree?

15. The radius of a bicycle wheel is 14 in. What is its circumference?

16. The circumference of a wheel is 88 in. What is its diameter?

17. State the steps necessary to take to find the area of a circle when you know its circumference.

18. What is π ?

19. State the dimensions of a cord of wood.

20. Without finding the number of cubic feet, find the number of cords in a pile of wood 16 ft. long, 4 ft. wide, and 4 ft. high.

1. Find the approximate difference in area between a field 40 rd. square and one 40 rd. in diameter. Find the exact difference.

2. Find the approximate difference in volume between two cisterns, one in the form of a cylinder 6 ft. in diameter and 6 ft. deep, and the other in the form of a 6-ft. cube.

3. What is the cost of digging a cistern in the form of a cylinder whose diameter is 6 ft., and whose depth is 18 ft., at 62¢ a cubic yard? How many gallons of water will this cistern hold?

4. A house is to be built $40' \times 30' \times 20'$. Estimate the number of bricks required for the walls, if the walls are 1 ft. in thickness and no allowance is made for openings.

5. The cellar of this house is 6 ft. deep and the walls are 16 in. thick, how many cubic feet of stone will be needed for the cellar wall?

6. At 18¢ a pound, $4\frac{1}{2}$ lb. to a square foot, find the cost of lead to line a tank 6 ft. by 5 ft., and $4\frac{1}{2}$ ft. deep.

7. How many bushels of potatoes can be put in a wagon-box 18 ft. long, 3 ft. wide, and 2 ft. deep?

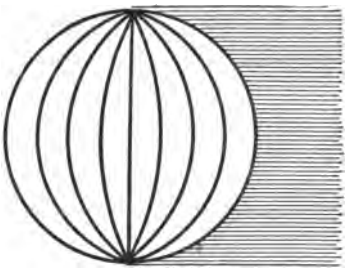
8. Find the cost of the following bill of lumber at \$18 per M:

200 boards, 16 ft. by 10 in. by 1 in.
150 boards, 12 ft. by 12 in. by 1 in.
40 planks, 12 ft. by 10 in. by 3 in.
20 scantling, 15 ft. by 4 in. by 2 in.

9. What will it cost to paper a room $24' \times 20' \times 10'$ with paper costing 20¢ a roll? There are 2 windows $5' \times 4'$ and a door $6' \times 4\frac{1}{2}'$.

10. At \$175 an acre, what is the value of a triangular piece of land whose base is 64 rd. and the perpendicular distance from the base to the vertex is 32 rd.?

11. How many square inches of tin will be required to make 50 boxes 9 in. long, 4 in. wide, and $3\frac{1}{2}$ in. deep?



1. From the globe or from this diagram notice the lines that extend from the North to the South Pole.

2. These lines are called **meridians**, that is the *lines of midday*.

3. We usually call the meridian that passes through the Royal Observatory at Greenwich, England, the *prime meridian*.

4. Distance east or west of the prime meridian measured in degrees is called **longitude**.

5. Repeat the table of circular measure. How many degrees are there in the circumference of the earth?

6. How long a time is required for the earth to rotate on its axis?

7. Every point upon the surface of the earth passes through 360° in 24 hours. Since this is true we say 24 hours of time correspond to 360°.

8. One hour of time corresponds to how many degrees? What is $\frac{1}{24}$ of 360°?

9. If 1 hour of time corresponds to 15°, then 1 min. of time corresponds to $\frac{1}{60}$ of 15° = $\frac{1}{4}$° = 15′. In the same way we find that 1 sec. of time corresponds to 15″ of longitude.

10. In what direction does the earth revolve? In what direction, then, does the sun *appear* to move?

11. Do places east or west of us see the sun rise first?

12. If the places east of us begin the day earlier, will they have earlier or later time than we have?

Learn: Places east of us have later time.
Places west of us have earlier time.

13. A is situated 15° east of B. When it is 12 M. at B, what time is it at A?

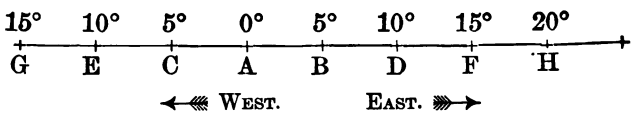
1. What is the difference in time of two places if the difference in longitude is 15°? 30°? 60°? 90°?

2. State a rule for finding the difference in time when the difference in longitude is known.

3. What is the difference in longitude between two places if the difference in time is 1 hour? 3 hours? $2\frac{1}{2}$ hours?

4. State a rule for finding the difference in longitude, when the difference in time is known.

5. When it is noon here, what time is at it a place 15° east of here? 30° west? 120° west? 90° east? 45° east? 15° 30′ west? 30° 15′ east?

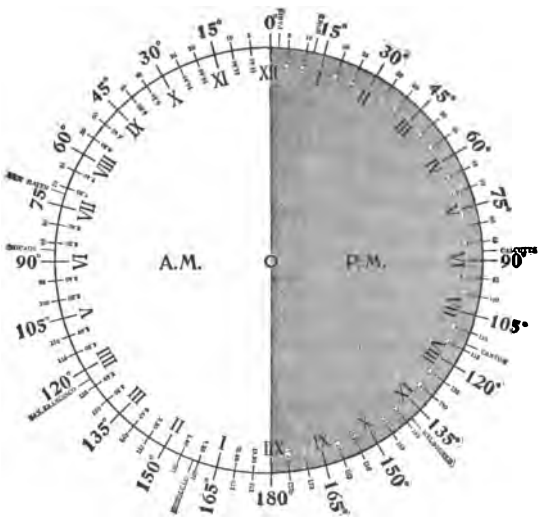


6. How far apart are A and D? How was it found?
7. How far apart are B and F? How was it found?
8. How far apart are A and C? How was it found?
9. How far apart are C and G? How was it found?
10. How far apart are B and C? How was it found?
11. How far apart are D and G? How was it found?

12. State a rule for finding the difference in longitude between two places when both are in east longitude? When both are in west longitude? When one is in east and the other in west longitude?

Find the difference in longitude between places situated at the following meridians:

13. 15° W. and 45° W.
14. 15° E. and 60° E.
15. 20° E. and 50° E.
16. 30° W. and 75° W.
17. 15° E. and 30° W.
18. 30° W. and 45° E.
19. 40° W. and 20° E.
20. 25° E. and 35° W.
21. 24° E. and 6° W.
22. 32° W. and 28° E.



This drawing was made from a chart used in solving problems in Longitude and Time.

DIRECTIONS FOR MAKING

a. Cut a circle having a 4-in. radius, and divide it into degrees.

b. Cut a circle having a 3½-in. radius, and divide it into 24 equal parts, to correspond with the hours in the day.

c. Fasten these two circles by an eyelet at the center so that one can revolve upon the other.

1. When it is 2 P.M. at A, 60° west longitude, what time is it at B, 90° east longitude?

A. Solution by using chart:

Turn the hour dial until 2 P.M., the time at A coincides with 60° W., the longitude of A. Find 90° E., the longitude of B, and on the hour dial read the corresponding time.

B. 60° + 90° = 150°.
150° ÷ 15° = 10. ∴ 10 hr.
2 P.M. + 10 hr. = 12 midnight.

First find the difference in longitude. Change it to difference in time by dividing by 15°. The difference in time is 10 hr. Since B, 90° E., is *east* of A, 60° W., its time must be *later*, and 10 hr. later than 2 P.M. is 12 midnight.

TABLE OF LONGITUDES

Amsterdam, 4° 53′ 4″ E.	New Orleans, 90° 3′ 28″ W.
Berlin, 13° 23′ 43″ E.	New York, 74° 0′ 3″ W.
Boston, 71° 3′ 30″ W.	Paris, 2° 20′ 22″ E.
Canton, 113° 16′ 30″ E.	Rome, 12° 27′ 14″ E.
Chicago, 87° 36′ 45″ W.	San Francisco, 122° 26′ 48″ W.
Manila, 120° 58′ 6″ E.	Washington, 77° 3′ 6″ W.

When it is noon at Paris, what time is it at Boston?

$$
\begin{array}{r}
2^\circ\ 20'\ 22'' \\
71^\circ\ 3'\ 30'' \\
\hline
15)\overline{73^\circ\ 23'\ 52''} \\
4\ \ 53\ \ 35
\end{array}
$$

$\therefore$ 4 hr. 53 min. 35 sec.

$$
\begin{array}{r}
12 - 0 - 0 \\
4 - 53 - 35 \\
\hline
7 \text{ hr. } 6 \text{ min. } 25 \text{ sec.}
\end{array}
$$

One place being in east longitude and the other in west longitude, we add and find the difference to be 73° 23′ 52″. Since 15° of longitude makes a difference of 1 hr. in time, and 15′ of longitude makes a difference of 1 min. in time, we divide the difference in longitude by 15°, and find the difference in time to be 4 hr. 53 min. 35 sec. Since Boston is *west* of Paris, it must have *earlier* time, so we subtract 4 hr. 53 min. 35 sec. from 12 M. and find the time is 6 min. 25 sec. past 7 o'clock A.M.

When it is noon at New York, what is the solar time of:

1. Amsterdam?
2. New Orleans?
3. Berlin?
4. Boston?
5. Paris?
6. Rome?
7. Canton?
8. Chicago?
9. San Francisco?

10. When it is 3 P.M. at New York, what time is it in New Orleans?

11. When it is noon at Canton, what time is it at Washington?

12. When it is 6 A.M. at Chicago, what time is it at Rome?

13. When it is 9 A.M. at C, 120° W., it is 12 M. at D. Find the longitude of D.

A. Turn the hour dial until 9 A.M. coincides with 120° W., Find 12 M. and read the longitude on the corresponding dial.

B. 12 hr. − 9 hr. = 3 hr.
$3 \times 15^\circ = 45^\circ$
$120^\circ - 45^\circ = 75^\circ$ W.

First find the difference in time. Since 1 hr. in time makes a difference of 15°, we multiply 15° by 3 and find the difference in longitude is 45°. Since the time at D is *later* than the time at C, it must be east of it. When we are in west longitude and travel east, we are moving toward the prime meridian, and therefore subtract 45° from 120°.

14. When it is noon at Paris, what is the time at St. Petersburg, 30° 16′ E?

When it is 10 o'clock A.M. at Philadelphia, 75° 10′, it is 10 min. past 3 P.M. at Paris. What is the longitude of Paris?

15 hr. 10 min.	77° − 30′
10 hr.	75° − 10′
5 hr. 10 min.	2° − 20′ E.
15	
75 − 150	
77° 30′	

3 P.M. is the 15th hour of the day. The difference in time is 5 hr. 10 min., which corresponds to 77° 30′ difference in longitude. Since Paris has *later* time than Philadelphia it must be 77° 30′ *east* of it. Since the prime meridian is only 75° 10′ east of Philadelphia, then Paris must be the difference between 77° 30′ and 75° 10′ east of the prime meridian.

1. A and B sailed together from New York. A kept his watch by New York time and B set his by the sun every day. In a few days the watches were 2 hr. 16 min. apart. Whose watch was the faster? In what longitude were they?

2. The longitude of Jerusalem is 35° 32′ E. When it is 7.30 A.M. at New York, what time is it at Jerusalem?

3. What change must a person make in his watch in going from Washington to Paris?

4. A is 45° 10′ east of B. What time is it at B, when it is 6 o'clock P.M. at A?

5. When it is noon at Greenwich it is 5 hr. 50 min. 27 sec. earlier at another place. From the table of longitudes, find the name of the other place.

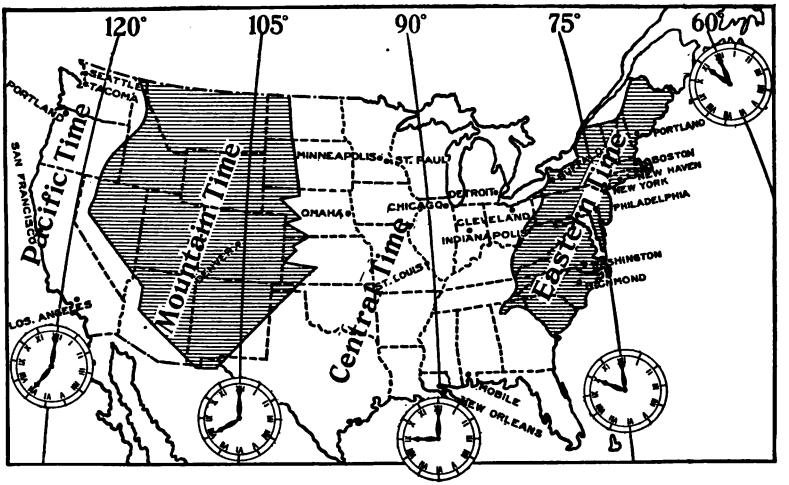
6. Suppose an account of an accident, that happened in Manila 120° E. at 5 A.M. Wednesday morning, was sent to New York with only a loss of one hour in transmission, at what time would it reach New York?

7. At what time will quotations of the stock exchange in Paris reach New York, if cabled at 1 P.M.?

8. A ship in distress reports by wireless telegraph that she is in longitude 45° 30′ west. If the message was sent at 10 A.M., when could the Boston station catch the message?

9. Allowing no time for transmission when could the New York papers receive dispatches sent at noon from Canton? Rome? Manila?

Standard Time



1. The difference in time between all places caused such inconvenience to the railroads and persons traveling that in November, 1883, the railroads of the United States and Canada adopted what is called Standard Time.

2. Four meridians were chosen, 15° apart, as central meridans. These are the 75°th, 90°th, 105°th, 120°th. It was intended that all places $7\frac{1}{2}$° on either side of these meridians should form a belt, but as a matter of fact the railroads found it necessary to change time at some convenient point. The result is very irregular boundaries for the belts.

3. These time belts are called Eastern, Central, Mountain, and Pacific.

4. When it is noon in the Central belt, what time is it in the Eastern? Mountain? Pacific?

1. In traveling from Boston to Minneapolis, what change will a traveler need to make in his watch?

2. What changes must a man make in his watch in traveling from San Francisco to Boston?

3. A train arrives in Buffalo at 1.15 P.M. It waits 15 min. and leaves at 12.30 P.M. Explain how this can be true.

4. When it is 12 M. at Baltimore, what time is it at San Francisco? At Denver? At New Haven?

5. How should a watch be changed in going from Connecticut to Minnesota? From California to Ohio?

6. What is the difference between the true time and the standard time at a place whose longitude is 85° W.? 125° W.

Give the difference in standard time, and tell which place has earlier time:

7. Boston and St. Louis.

8. Washington and Tacoma.

9. Portland, Me., and Seattle.

10. Richmond and Omaha.

11. Philadelphia and Chicago.

12. Los Angeles and St. Paul.

13. Mobile and Portland, Or.

14. How should a watch be changed in going from New York to St. Louis? From Chicago to Washington?

15. When it is 1 P.M. at Baltimore, what time is it at Indianapolis? At Denver?

16. A panic on the stock exchange in New York was reported at 1 P.M. by telegraph to Cleveland, San Francisco, Boston, New Orleans, Minneapolis, and Buffalo. Allowing 10 min. for transmission, when did the message reach each city?

17. When it is 2 P.M. in Texas, what time is it in Maine? California? Florida? Minnesota?

1. How many feet board measure are there in 48 boards, $\frac{7}{8}''$ thick, 4″ wide, and 15′ long?

2. Change to whole or mixed numbers: $\frac{242}{69}$, $\frac{153}{55}$, $\frac{196}{13}$, $\frac{551}{17}$.

3. Change to improper fractions: $47\frac{2}{3}$, $96\frac{11}{13}$, $13\frac{7}{19}$, $41\frac{27}{100}$.

4. Find the product of $\frac{14}{50} \times \frac{75}{91}$. $22 \times 6\frac{7}{11}$. $\frac{7}{8} \times \frac{8}{15} \times \frac{25}{2}$.

5. $89\frac{3}{5}$ is $\frac{14}{25}$ of what number?

6. Find the sum of $\frac{8}{15}$, $\frac{7}{10}$, $\frac{5}{8}$, $\frac{6}{25}$.

7. From $41\frac{23}{36}$ take $11\frac{7}{8}$.

8. A can do a piece of work in 15 days, B in 18 days, and C in 12 days. How many days will it take the three working together to do the work?

9. When it is 9 o'clock P.M. Wednesday, in Chicago, what time is it in Berlin?

10. A commission merchant sold 7500 bu. of wheat at 65¢ and charged $2\frac{1}{2}\%$ commission. How much money should he remit to his employer?

11. How many 2-ft. cubes can be cut from a block 8′ × 6′ × 4′, if no allowance is made for waste in cutting?

12. What is the ratio of 191 lb. 10 oz. to 27 lb. 6 oz.?

13. $28.50 was deducted from the list price of an article. If the discount was $37\frac{1}{2}\%$, what was the list price?

14. A farmer had a field 24 yd. long and 18 yd. wide, in which he planted potatoes. The rows ran lengthwise and were 2 ft. apart, and 2 ft. from the fence. The hills were 18 in. apart, and 18 in. from the fence. How many hills of potatoes were planted?

15. Add 215 units, 215 tenths, 215 hundredths, and 215 thousandths.

16. In the center of a circular field 30 rd. in diameter is a circular pond 10 rd. in diameter. How many square rods of the field are not covered by water?

17. What per cent of $2\frac{1}{2}$ A, is 40 sq. rd.?

1. If 5 bbl. of flour cost \$20, what will 25 bbl. cost?

2. The divisor is 7¢, the quotient 9, and the remainder 2¢. What is the dividend?

3. The remainder is \$15, and the subtrahend \$42. What is the minuend?

4. The product is \$80 and the multiplier is 5. What the multiplicand?

5. The product is \$120 and the multiplicand \$40. What is the multiplier?

6. The divisor is 15, the quotient is \$5, and the remainder \$2. What is the dividend?

7. If 9 yd. of muslin cost \$1.08, what will 7 yd. cost?

8. How many pecks are there in 6 bu. 3 pk.?

9. Change 4 hr. 35 min. to minutes.

10. Change 4° 20′ to minutes.

11. A city lot contains 6000 sq. ft. If it has a 50-ft. front, how deep is the lot?

12. A room is 15 ft. by 20 ft. What is its perimeter? If the room is 10 ft. high how many square feet are there in its walls? How many square feet are there in the walls and ceiling?

13. A box that is 10 in. long and 4 in. wide, contains 240 cu. in. How deep is the box?

14. The sum of two numbers is 35 and the smaller is 15. What is the larger number?

15. How many days are there from May 5 to June 15?

16. A man receives \$2.40 a day and his boy $\frac{1}{4}$ as much. What will both earn in 2 weeks?

17. An automobile averages 15 mi. an hour. How far will it go in 5 days of 10 hr. each?

18. When it is noon at New Haven, what time is it at a place 45° east of it? 60° west? 90° east? 120° west?

19. Change: $7\frac{3}{4}$ to fourths; $3\frac{3}{8}$ to eighths; $12\frac{1}{3}$ to thirds.

1. \$4620 is 131% more than another number.

2. An article that cost \$20,000, was sold for \$19,700.

3. A and B can do a piece of work in 18 days. A can do it alone in 24 days.

4. A real estate agent sold two houses at \$3900 each. On one he made 20% and on the other he lost 25%.

5. The diameter of a cylinder is 16 in. and its length is 12 ft.

6. The difference between two numbers is 40, and the larger is 5 times the smaller.

7. A man had \$5000 in the bank. He drew out 20% of it, then 30% of the remainder. He afterwards deposited 15% of the sum he had drawn out.

8. A man loaned his friends \$900. At the end of 3 yr. 4 mo. 27 d., the friend returned it with interest at 5%.

9. A man insured his house for \$3500 at $1\frac{3}{4}\%$, and his furniture for \$3000 at $\frac{3}{4}\%$.

10. The base of a rhomboid measures 42 ft. and its altitude 16 ft.

11. A merchant imported from France 24 doz. pairs of gloves, invoiced at 6 francs a pair.

12. A wholesale dealer sold 3400 bbl. of flour at \$5.00 a barrel with a discount of 8% and $2\frac{1}{2}\%$.

13. A rectangular field is 215.11 ft. long, and its length is 7 times its width.

14. A man bought a horse, carriage, and harness for \$240. He gave three times as much for the carriage as for the harness and as much for the horse as he did for both the carriage and the harness.

15. A half mile of road was built at a cost of \$4.75 a rod.

16. In the center of a room, $23\frac{1}{2}$ ft. square, there is a rug 18 ft. square. The rest of the floor is covered with oilcloth, which extends 6 in. under the rug on all sides. The rug cost \$1.75, and the oilcloth 85¢ a square yard.

1. Suppose you each put \$200 in a savings bank Jan. 1, with the understanding that you are to receive 4% interest. How much interest will be due you the following January?

2. If you go to the bank and draw out \$8, you will still have in the bank how much money?

3. If you do not draw out the \$8, it will be added to the sum you first put in. How much money will you have on interest the second year?

4. What will be the interest of \$208 for the second year?

5. What will be done with the \$8.32, if it is not called for? How much will you have in the bank the third year?

6. How much more money have you in the bank at the end of the third year than you put in the bank at first?

Solution:

\$200.00 Principal.
8.00 Interest for 1st year.
\$208.00 Principal for 2d year.
8.32 Interest for 2d year.
\$216.32 Principal for 3d year.
8.65 Interest for 3d year.
\$224.97 Amount in bank.
200.00 1st Principal.
\$24.97 Interest for 3 years.

Find the amount of the first principal for the first period of time. Use this amount as a new principal and find the amount for the second period of time, and so continue. The last amount is called the compound amount. From the last amount subtract the first principal to find the compound interest.

7. When the interest as it becomes due is added to the principal, and the interest is reckoned on the amount, the owner is said to receive **compound interest.**

8. Interest is usually compounded annually, semi-annually, or quarterly. In this book compound annually, unless otherwise directed.

1. What is the compound interest on \$450 for 4 yr. 3 mo. 15 d. at 4%?

NOTE:—First find the amount for the whole periods, 4 yr. Then find the interest on this amount for the remaining time, 3 mo. 15 d.

2. What is the difference between the simple and compound interest on \$500 for 5 yr. 6 mo. 18 d. at 6%?

Find the compound interest on:

3. \$4500 for 2 yr. 6 mo. at $3\frac{1}{2}$%.
4. \$1278 for 3 yr. 9 mo. at 4%.
5. \$2576 for 4 yr. 2 mo. 12 d. at 5%.
6. \$728 for 2 yr. 9 mo. 24 d. at 3%.
7. \$6793 for 3 yr. 6 mo. 15 d. at 4%.
8. Find the compound interest on \$600 for 1 yr. 9 mo. at 4% annually, compounded semi-annually.

\$600	1st Principal.
12.00	Int. for 1st period.
\$612.00	2d principal.
12.24	Int. for 2d period.
\$624.24	3d principal.
12.484	Int. for 3d period.
\$636.724	4th principal.
6.367	Int. for 3 mo.
\$643.091	Compound amount.
600.00	1st principal.
\$43.09	Compound interest.

Since the rate is 4% compounded semi-annually, there will be 3 full periods of 6 mo. each, and the rate for each period will be 2%. This gives us \$636.724 as our amount. Find the simple interest on this sum for the remaining 3 mo. at 4%.

Find the compound interest on:

9. \$1678 for 2 yr. 5 mo. at 6%, compounded semi-annually.
10. \$550 for 1 yr. 4 mo. at 4%, compounded quarterly.
11. \$1248 for 4 yr. 6 mo. at 4%, compounded semi-annually.
12. \$4124 for 3 yr. 3 mo. at 6%, compounded quarterly.

Bankers always find both simple and compound interest by means of tables.

COMPOUND INTEREST TABLE

AMOUNT OF $1, AT VARIOUS RATES, COMPOUND INTEREST, 1 TO 20 YEARS

Years	1½%	2%	2½%	3%	4%	5%	6%
1	1.015000	1.020000	1.025000	1.030000	1.040000	1.050000	1.060000
2 ...	1.030225	1.040400	1.050625	1.060900	1.081600	1.102500	1.123600
3 ...	1.045678	1.061208	1.076891	1.092727	1.124864	1.157625	1.191016
4 ...	1.061364	1.082432	1.103813	1.125509	1.169859	1.215506	1.262477
5 ...	1.077284	1.104081	1.131408	1.159274	1.216653	1.276282	1.338226
6 ...	1.093443	1.126162	1.159693	1.194052	1.265319	1.340096	1.418519
7 ...	1.109845	1.148686	1.188686	1.229874	1.315932	1.407100	1.503630
8 ...	1.126493	1.171659	1.218403	1.266770	1.368569	1.477455	1.593848
9 ...	1.143390	1.195093	1.248863	1.304773	1.423312	1.551328	1.689479
10 ...	1.160541	1.218994	1.280085	1.343916	1.480244	1.628895	1.790848
11 ...	1.177949	1.243374	1.312087	1.384234	1.539454	1.710339	1.898299
12 ...	1.195618	1.268242	1.344889	1.425761	1.601032	1.795856	2.012197
13 ...	1.213552	1.293607	1.378511	1.468534	1.665074	1.885649	2.132928
14 ...	1.231756	1.319479	1.412974	1.512590	1.731676	1.979932	2.260904
15 ...	1.250232	1.345868	1.448298	1.557967	1.800944	2.078928	2.396558
16 ...	1.268986	1.372786	1.484506	1.604706	1.872981	2.182875	2.540352
17 ...	1.288020	1.400241	1.521618	1.652848	1.947901	2.292018	2.692773
18 ...	1.307341	1.428246	1.559659	1.702433	2.025817	2.406619	2.854339
19 ...	1.326951	1.456811	1.598650	1.753506	2.106849	2.526950	3.025600
20 ...	1.346855	1.485947	1.638616	1.806111	2.191123	2.653298	3.207136

1. Find the compound interest on $700 for 14 yr. 4 mo. at 4%.

From the table we find that the amount of $1 at 4% compound interest for 14 yr. is $1.731676. The amount of $700 for the same time and rate must be 700 times $1.731676 = $1212.173. Find the amount on this sum for 4 mo. at 4%.

2. Find the compound interest on $2200 for 15 yr. 6 mo. at 5%.

3. Find the compound interest on $480 for 9 yr. at 5%, compounded semi-annually. This is the same as for 18 yr. at 2½%.

4. Find the compound interest on $1850 for 5 yr. 6 mo. at 6%, compounded quarterly. This is the same as for 22 yr. at 1½%.

Find the compound interest of:

1. \$378 for 3 yr. 4 mo. at 3%.
2. \$2718 for 2 yr. 8 mo. at 4%.
3. \$5267 for 3 yr. 2 mo. 12 da. at 5%.
4. \$1700 for 2 yr. 8 mo. 20 da. at $3\frac{1}{2}$%.
5. \$500 for 2 yr. 6 mo. at 4%, compounded semi-annually. Since the interest is compounded semi-annually, there will be 5 periods, and the rate will be 2%.
6. \$400 for 1 yr. 9 mo. at 6%, compounded quarterly. In 1 yr. 9 mo. there are 7 quarterly periods, and the rate will be $\frac{1}{4}$ of 6% or $1\frac{1}{2}$%.
7. \$1600 for 3 yr. 3 mo. at 5%, compounded semi-annually.
8. \$1200 for 2 yr. 6 mo. at 4%, compounded quarterly.
9. \$900 for 1 yr. 3 mo. at 4%, compounded quarterly.
10. \$1500 for 3 yr. 9 mo. at 6%, compounded semi-annually.

Bankers usually find both simple and compound interest by means of tables. The compound interest table may be found on page 249.

11. Find the compound interest of \$900 for 12 yr. at 3%.

By the table we find that the amount of \$1 at 3% compound interest for 12 yr. is \$1.425761. The amount of \$900 for the same time and rate must be 900 times \$1.425761 = \$1284.184. \$1284.184 − \$900 = \$384.18.

12. Find the compound interest of \$1200 for 4 yr. 9 mo. at 6%, compounded quarterly. This is the same as the interest of \$1200 for 19 yr. at $1\frac{1}{2}$%.

Find by the table the compound interest of:

13. \$2000 for 15 yr. at 5%.
14. \$1600 for 14 yr. at 4%.
15. \$2400 for 12 yr. at 3%.
16. \$1800 for 8 yr. 6 mo. at 5%, compounded semi-annually.

1. Mention several things for which a city or town needs money.

2. Money to pay for schools, roads, sewers, lights, fire and police protection, is raised by means of a tax.

3. A sum of money assessed on persons and owners of property to meet the expenses of a town, county, city, or state, is called **a tax.**

4. **Poll** means head. What, then, is a poll-tax?

5. Who pays a poll-tax? Does a man need to own property in order to be assessed a poll-tax?

6. Any tax upon real estate or personal property is called **property tax.**

7. **Real estate** is a form of property that consists of lands, houses, or, in general, immovable property.

8. **Personal property** consists of money, horses, merchandise, or, in general, movable property.

9. **Assessors** are persons appointed to make an inventory of all taxable property and estimate its value. The value the assessors put on property is called its **assessed value.** In some places property is not assessed for its full value.

10. The **collector** is the one chosen to collect the taxes.

11. Find out how large a poll-tax is assessed in your town or city, and also how large a tax on property is assessed.

12. Taxes are usually assessed as follows:

a. The state determines the amount to be expended for state purposes and divides that amount among the counties according to their valuation.

b. The county adds to this sum the amount it will need for county expenses and divides the total among the towns of the county according to their valuation.

c. Each town adds to this amount whatever it needs for its own local expenses and thus finds the total tax or tax levy.

1. In some states the county and state tax are found separately and are not united with the town tax.

2. The **rate of taxation** is usually expressed as a certain amount on each dollar, which can be easily changed to a rate per cent. Thus a tax of 15 mills on a dollar is $1\frac{1}{2}\%$.

3. Taxes are usually collected as follows:

a. The assessors find the number who must pay a poll-tax. Then multiply the tax on one poll by the number of polls to find the *poll-tax.* The poll-tax is then subtracted from the *tax levy.* The remainder is the *property tax.*

b. The assessors next find the entire valuation of the town, which is called the *total valuation.*

c. They next divide the property tax by the total valuation to find the tax on \$1, called the *tax rate.*

d. They now multiply the value of each man's property by the tax rate and find his tax on property.

4. There are 4120 persons, each of whom pays \$1.50 in the city of B. The total valuation is \$5,864,528, and the total tax \$94,147.92. What property tax must Mr. Philips pay who owns a mill valued at \$4500 and who owns personal property at \$1240?

$\$1\frac{1}{2} \times 4120 = \$6180.$
$\$94{,}147.92 - \$6180 = \$87{,}967.92.$
$\$87{,}967.92 \div \$5{,}864{,}528 = .015.$
$\$4500 + \$1240 = \$5740.$
$\$5740 \times .015 = \$86.10$

Study each step as given in example 3, and compare with each step of this solution.

5. The property valuation of a town is \$1,500,000, and the tax levy is \$12,500. There are 250 men, each paying \$2. Mr. Dunbar's real estate is valued at \$7500 and his personal property at \$4500. Find his tax, including a poll-tax.

6. In the town of C there are 2500 polls, each taxed \$2. The tax levy is \$245,000, and the taxable property is \$12,000,000. Find Mr. J's tax on property worth \$125,000.

1. Given the amount of taxable property and the tax to be raised, what can you find? How can you find it?

2. The amount of taxable property is \$200,000,000. If the tax levy is \$2,000,000, what is the tax rate?

3. Given the amount of taxable property and the rate of taxation, what can you find? How can you find it?

4. Mr. S is worth \$50,000. If the rate of taxation is $1\frac{1}{2}\%$, what tax must Mr. S. pay?

5. Given the tax to be raised, and the rate of taxation, what can you find? How can you find it?

6. Mr. M's tax last year was \$45. If the rate of taxation was 15 mills on \$1, find the assessed valuation of his property.

7. The rate of taxation in the town of C is $1\frac{1}{2}\%$. What is the tax of Mr. A, whose real estate is valued at \$3000 and personal property at \$1000?

8. When the rate of taxation is $1\frac{3}{5}\%$, what tax must Mr. Smith pay, if he is worth \$5000?

9. Find the tax on property assessed at \$2000, when the rate of taxation is 12 mills on a dollar.

10. Mr. A paid \$6 as a special tax levied at 6 mills on \$1. Find the value of his property.

11. The tax rate is 13 mills. How much tax must be paid on property assessed at \$12,000?

12. In a certain town there are 1200 polls, each assessed \$2. Find the poll-tax. If the tax levy is \$40,000, how large a tax must be raised on property?

13. In the town of B, the assessed valuation of the property is \$200,000, and the tax levy is \$4000. What is the rate of taxation?

14. If the tax rate is $1\frac{1}{2}\%$ and your father's property is assessed at \$4000, what tax must he pay, including a poll-tax of \$2?

1. A tax of \$50,000 is levied in a town valued at \$3,200,000. There are 1000 persons who pay a poll-tax of \$2. What is C's tax if his property is valued at \$9000 and he pays a poll-tax?

2. The real estate of a city is assessed at \$3,099,500, and the personal property at \$1,487,280. The tax levy is \$66,023.92, and there are 1340 polls assessed at \$1.35. Find Mr. A's poll and property tax, if his property is assessed at \$18,000.

3. When the tax rate is 21 mills on a dollar, what is the assessed value of property that pays a tax of \$626.22?

4. What is the rate of taxation when \$2,285.10 is the tax upon \$152,340?

5. There are 200 taxable polls in the town of Sheffield, each assessed \$2. The expenses of the town are \$16,150, and the valuation of the property is \$1,750,000. Mr. Mellen owns \$2460 worth of property. Find his tax including one poll.

6. A man's property is assessed at \$6500 and the rate is 3 mills on a dollar. Find the amount of his tax if he is allowed 2% for payment before a certain date.

7. In the town of H there are 1200 polls, each taxed \$1. The taxable property is \$30,000,000 and the tax levy is \$151,200. What tax does Mr. Sims pay who is assessed \$12,000 for real estate and \$2500 for personal property and who pays one poll-tax?

8. How much tax will a person pay whose property is assessed at \$246,500, if he pays $1\frac{3}{4}\%$ city tax, 1% school tax, and $\frac{3}{4}\%$ county tax?

9. The real estate of a town is assessed at \$1,105,843, and the personal property at \$249,031. The tax to be raised is \$21,258.11, and there are 850 polls, each assessed \$1.10. What tax will Mr. Martin pay if his property in the town is assessed at \$8650 and he, not being a resident of the town, pays no poll-tax?

1. What dimensions of a triangle do you need to know to find its area?

2. How do you find the perimeter of a triangle?

3. What dimensions of a square or a rectangle must be known to find its area?

4. Given the perimeter of a square, how do you find its area?

5. Given the perimeter and length of a rectangle, how do you find its width? How do you find its area?

6. What is a parallelogram? What dimensions do you need to know to find its area?

7. Given the area and width of a parallelogram, what can be found? How can you find it?

8. Given the area of a triangle and its altitude, what can you find? How can you find it?

9. Given the diameter of a circle, how can you find its area?

10. Given the circumference of a circle, how can you find its area?

11. What 3 things do you need to know to find out the number of yards of carpet required for any room?

12. What 3 things do you need to know to find out how many square feet of plastering are needed for any room?

13. What dimensions of a rectangular solid do you need to know to find its volume?

14. Given the volume and two dimensions of a rectangular solid, how do you find the other dimensions?

15. Given the height and diameter of a cylinder, what can you find? How can you find it?

16. Given the volume and diameter of a cylinder, what can you find? How can you find it?

17. How would you estimate the number of bricks needed for a straight wall? Needed for a house?

1. Place a cube on your desk. How many dimensions has it? How many faces has it?

2. These faces are called its surfaces. A **surface** is a boundary of a solid. Define surface.

3. How many dimensions has each surface?

4. Surfaces are bounded by edges called lines. A **line** is the limit of a surface, or it is the path traced by a point as it moves from one position to another. To read a line we usually use two letters, naming the starting-point first.

5. How many dimensions has a line?

6. How are the lines of a cube limited?

7. A **point** is the limit of a line and has no extent, only position. Define point.

8. In the cube how many faces meet to form a line?

9. Each face is bounded by how many lines? If the cube has six faces, and each face has four lines, how many lines has the cube? Why is not the number 24?

10. How many lines meet at each point? If the cube has 12 lines, and each line has two points, how many points has the cube? Why not twenty-four?

11. In a square prism, how many surfaces, lines, and points are there?

12. By how many lines is each surface bounded? How many surfaces meet in each line? How many lines meet at each point? Are the surfaces the same shape and size?

13. Examine in the same way a triangular prism and an hexagonal prism.

14. What kind of lines have you found on these solids?

15. A **straight line** is a line which has the same direction throughout its entire length.

16. Define a straight line. Draw one.

17. Look at a cylinder. How many edges or lines has it? Are these lines straight? What are they?

1. Define a curved line. Write: A **curved line** is a line that constantly changes its direction.

2. Fasten a weight to one end of a cord. Hold the other end at rest on the hand. This is a **plumb line,** and is said to be vertical.

3. Define a vertical line. Draw one.

4. A **horizontal line** is a line which has the direction of any line in the surface of still water. Practically it is a line that points towards the horizon.

5. Lines neither vertical nor horizontal are called **inclined lines** or **oblique lines.**

6. How are horizontal lines represented on paper? Vertical lines?

7. Hold your ruler vertically, horizontally, inclined. Draw on paper lines to represent these three positions.

8. Draw a vertical line, and through it two horizontal lines.

9. Draw two lines which have the same direction, that is, do not meet, however far extended. These lines are said to be **parallel.** Define parallel lines.

10. Draw two parallel straight lines. Two parallel curved lines.

11. Draw two parallel horizontal lines. Two parallel vertical lines.

12. Draw two lines not parallel. Prolong them till they meet.

13. This point of meeting is called their **intersection.** Define intersection.

14. Hold two pencils parallel. Hold them so they will intersect.

15. Are two vertical lines always parallel to each other?

16. Can two horizontal lines ever intersect each other?

17. Give examples of vertical and horizontal lines.

1. Draw several parallel lines freehand. Test and correct them with ruler and triangle.

2. Write: When one line meets another line so as to make adjacent angles equal, the lines are said to be **perpendicular** to each other.

3. Draw two lines so as to form equal adjacent angles. Define perpendicular lines.

4. Draw a line perpendicular to a vertical line. Draw one perpendicular to an inclined line.

5. Draw three lines: (*a*) All parallel; (*b*) Two parallel, one perpendicular to them; (*c*) No two parallel, all intersecting at one point; (*d*) No two parallel, and not all meeting at a point.

6. In case (*d*), in how many points do the lines intersect?

7. Draw freehand a horizontal line of any length. Draw a vertical line of equal length. Test your work.

8. Draw two lines whose ratio shall be as 1 : 2; 1 : 4; 1 : 6.

9. Draw two lines whose ratio shall be as 2 : 3; 3 : 4; 2 : 6.

10. What is meant by drawing to a scale?

11. How long would you draw a line to represent 20 in., using a scale 1 : 8?

12. How many centers can a circle have? How many circles can have the same center?

13. Can a plane surface and a curved surface be parallel? Draw a straight line and a curved line that shall be parallel.

14. If I use a scale of $\frac{1}{4}$ in. to a foot, what ratio do I use?

15. What is the standard unit of length in this country?

16. Draw to a convenient scale lines representing 130 ft.; 250 yd.; 75 rd.

17. What scale did you use?

18. How many lines parallel to a given line can be drawn through a point outside of the line?

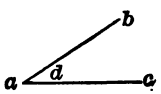
1. Take the dividers, and open the points one inch.

2. Do the legs of the dividers now point in the same direction or in different directions?

3. Draw lines to represent the legs of the dividers.

4. These lines are said to make an angle.

5. What is an angle? An **angle** is the difference in direction of two lines.



6. In this figure the lines *ab* and *ac* are called the sides of the angle. The point where these lines meet, as at *a*, is the **vertex.**

7. Define vertex.

8. The angle in 6 is named by reading the angle *d*, or the angle *cab*.

9. Draw two lines perpendicular to each other. The angle you have formed is called a right angle.

10. Define a right angle.

11. Draw two lines that shall meet but not be perpendicular to each other. The angle formed is an oblique angle.

12. Define an oblique angle.

13. Is the angle you have formed less than a right angle? If so, it is an acute angle.

14. Define an acute angle.

15. Is the angle you have formed greater than a right angle? If so, it is an obtuse angle.

16. Define an obtuse angle.

17. At 2 o'clock what angle do the hands of a watch make?

18. At 9 o'clock what angle do the hands make?

19. What kind of an angle does a vertical line make with a horizontal one?

20. Starting at 12 o'clock, in what time will the hands of a clock be at right angles to each other?

1. Draw several angles, and measure them by using a protractor. First estimate their size before measuring.

2. Using your protractor, make an angle of 45°; 60°; 90°; 30°; 150°; 135°; 100°; 40°; 20°.

3. Draw several angles, and by means of the protractor make other angles equal to each of them.

4. Draw an angle. Using protractor, make another angle twice the size of the given angle. Do the same, using dividers.

5. Draw an angle. Make another of $\frac{1}{2}$ the size; of four times the size.

6. Draw a horizontal line, AB. At a point C in the line, draw an oblique line that shall not cut AB. Measure the angles. Add their result. What is the sum?

7. Do the same after drawing one vertical and two oblique lines.

8. Draw three lines cutting each other at one point. How many angles are formed? Measure them. Add results. What is the sum? What ought it to be?

9. Draw a four-sided figure. Estimate the angles of the figure. Record your estimate in one column; the true value, found with a protractor, in another; and the error in a third column.

10. Substituting a five-sided figure, repeat 9.

11. Substituting a six-sided figure, repeat 9.

12. Using a protractor, erect a perpendicular at each extremity of a horizontal line; of an oblique line; of a vertical line.

13. Draw a vertical line. Using a protractor and ruler, construct a square upon this line.

14. How many degrees are passed over by the hour hand of a clock in an hour? In 3 hours?

15. In how many hours will the hour hand pass over 90°?

1. If 3 horses cost \$200, how many horses can be bought for \$1600?

2. $3\frac{1}{3}$ is what part of 10? $2\frac{1}{2}$ is what part of 10?

3. What is the ratio of 14 to $3\frac{1}{2}$?

4. If $3\frac{1}{2}$ yd. of cloth cost \$9, what will 14 yd. cost?

5. What is the ratio of 20 to $6\frac{2}{3}$?

6. If $6\frac{2}{3}$ yd. of ribbon cost 60¢, what will 20 yd. cost?

7. What is the ratio of 9 to 36?

8. If 36 lb. of sugar cost \$2.16, what will 9 lb. cost?

9. What is the ratio of \$2.80 to 35 cents?

10. If 8 qt. of peanuts cost \$.35, how many quarts can be bought for \$2.80?

11. $6 : 12 = 15 : x$. Find the missing term.

12. If 9 yd. of cloth cost \$25, what will 18 yd. cost?

13. If the antecedent of a ratio is 8, and the ratio is $\frac{1}{3}$, what is the consequent?

14. When it is 4 P.M. in New Haven, by standard time, what time is it in Chicago? In San Francisco?

15. When it is 10 A.M. in New York, it is 1 P.M. at another place. What is the difference in longitude? In which direction from New York is the second place?

16. Divide 90 into two such parts that one part shall be 4 times the other part.

17. What is the interest on \$240 for 3 mo. at 5%?

18. What is the interest on \$400 for 15 d. at $4\frac{1}{2}$%?

19. What is the premium on a policy of \$2400 for 3 yr. at $\frac{1}{2}$% a year?

20. A man's property is assessed at \$20,000. If the rate is 15 mills on a dollar, what tax must he pay?

21. The list price of a bill of goods is \$1000, with discounts of 20% and 5%. Find the net price of the goods.

22. If the discount in example 21 had been 25%, what would have been the difference in the net price?

1. A room measures 18′ × 12′ × 10′. At 37¢ a square yard find the cost of plastering it, allowing 80 sq. ft. for doors and windows. At $1.12½ a yard, find the cost of carpet, 27 in. wide, laid lengthwise, allowing 8 in. loss on each breadth for matching.

2. Find the cost of 10 girders, each 40 ft. long, 14 in. wide, and 12 in. thick, at $28 per thousand feet.

3. Find the interest on $3020 from Apr. 14, 1910, to June 10, 1913, at 5%.

4. Find the compound amount of $2000 for 4 yr. 6 mo. at 4%.

5. A merchant marked goods at 20% above cost and sold them at 12½% below the marked price. What per cent did he gain? How much did he gain on goods that cost $1200?

6. A man sold a span of horses for $660 and gained 10% by so doing. Find the cost. What would have been the loss per cent, if the horses had been sold for $570?

7. What is the rate of taxation when $147,960 is raised on real estate valued at $9,864,000?

8. In an election A received a certain number of votes. B received 35 more than twice as many, and C 75 less than 3 times as many. The total vote was 14,000. How many votes did each receive?

9. Mr. Jones has a city lot measuring 66 ft. by 132 ft. What is it worth at $2000 an acre?

10. A house valued at $7200, and insured for ¾ of its value at $12.50 on a $1000, was destroyed by fire. What was the owner's loss?

11. A can do a piece of work in 6 days, B in 8 days, and C in 10 days. How much of the work can they do in 2 days working together?

12. What is the circumference of a carriage wheel that makes 37½ revolutions in going 336 ft.

1. A **national bank**, also called a **commercial bank** and a **bank of deposit**, is an institution chartered by the national government, that is, given permission to do business.

2. The business that banks can do is:

a. To furnish a safe place of deposit for money.

b. To exchange money.

c. To issue notes for circulation.

d. To lend money on notes or any safe security, other than real estate.

e. To collect money on notes, drafts, and checks.

3. When a person opens an account at a bank, and each time he makes a deposit, he fills out a deposit slip and hands the slip and money to the *receiving teller*. The amount of money is entered in a small book, which the depositor keeps.

4. The depositor also receives a check book. By using these checks he can draw out money himself or pay any bill he may owe by making the check payable to the creditor, but withdrawals must not exceed the balance.

DEPOSITED BY

_____

IN

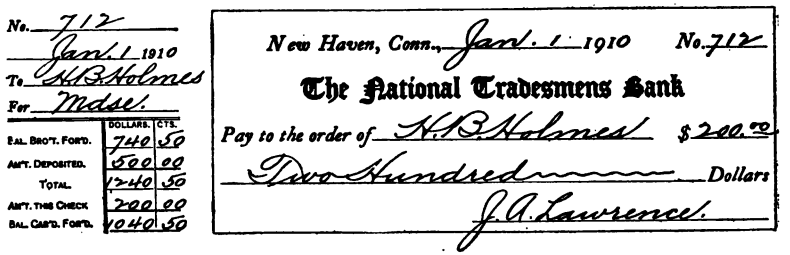
The National Tradesmens Bank

New Haven, _____ *19*__

	Dollars	Cents
Specie _____		
Bills _____		
Checks _____		
" _____		
" _____		
_____		

5. When a person wishes to draw money for himself, he usually writes the check payable to **self** or **bearer** or **cash**. It is better to use the first word, because if either of the other words is used, and the check is lost or stolen, any one can collect it.

6. A **check** is a written order on a bank for the payment of a certain sum of money to the person named on the check or to his order, and must be signed by the depositor.



No. 712
Jan. 1 1910
To H.B. Holmes
For Mdse.

	DOLLARS.	CTS.
BAL. BROT. FORD.	740	50
AMT. DEPOSITED.	500	00
TOTAL.	1240	50
AMT. THIS CHECK	200	00
BAL. CARD. FORD.	1040	50

New Haven, Conn., Jan. 1 1910 No. 712

The National Tradesmens Bank

Pay to the order of H.B. Holmes $200.00

Two Hundred Dollars

J.A. Lawrence.

1. The form above shows a page from a check book with the coupon which is left in the book after a check is torn out. This coupon shows in whose favor the check is drawn, the amount, the number, the date, and the balance left in the bank.

2. Suppose that J. A. Lawrence is owing $200 to H. B. Holmes of Chicago. Instead of sending the money he can fill out a check like the one above and send it to Mr. Holmes. Mr. Holmes must **indorse** it, that is, sign his name on the back. He may then deposit it and have it credited to his own account, or may have it cashed at any bank where he is known.

3. All banks have dealings with one another and the check will come back to the National Tradesmens Bank, where the amount will be deducted from Mr. Lawrence's balance.

4. When this check is canceled, and Mr. Lawrence's book has been balanced, it will be returned to him and can be kept by him as a receipt from Mr. Holmes, since he has indorsed it.

5. Make out a check for $73.50, payable to the order of S. H. Bruce. Suppose Mr. Bruce wished to make the check payable to W. E. Graves. He would write on the back, "Pay to the order of W. E. Graves," and sign his name beneath.

6. Make out a check for $45.15, payable to the order of Wm. W. Anthony. Have Mr. Anthony make the check payable to the order of Dr. E. N. Thorpe.

1. An important part of the business of a bank is loaning money on notes, because a large part of the capital employed in business is borrowed.

2. Suppose Mr. U. G. Blackmer sold H. E. Jordan an automobile for $1200, and instead of money received from Mr. Jordan a promissory note like the following, which he must pay in three months:

New Haven, Conn., Sept. 5, 1910 *No.* 114

Three *months after date* I *promise to pay*

to the order of U. G. Blackmer $1200

Twelve Hundred *Dollars*

at The National Tradesmens Bank *Due* _____

Value Received H. E. Jordan

3. Mr. Blackmer may keep this note until it is due, Dec. 5, '10, and then collect the money from Mr. Jordan. In that case no bank would be interested in it.

4. In case Mr. Blackmer wishes to receive money for the note at any time before it is due, he must:

a. Indorse it and thus become responsible for its payment, if Mr. Jordan should fail to pay it when due.

b. Present the note at the bank where he keeps his account, and if the officials are satisfied that both Mr. Blackmer and Mr. Jordan are reliable, they will accept the note and **discount** it, that is, advance some money.

5. In this note, who is the maker? Who is the payee? What is the face? What is the date? Is it a demand or time note?

6. Write a time note using the following data: — date, June 11, 1911; time, 60 days; face, $275; maker, G. C. Smith; payee, C. D. Brown.

1. **Bank discount** is the interest retained by a bank for advancing money on notes before they are due.

2. The **proceeds** is the amount received by the borrower, and is equal to the *face* of the note less the *discount.*

3. The **date of maturity** is the time when the note is due. It is found by adding the time specified in the note to the date. Thus, Sept. 5 + 3 mo. = Dec. 5.

NOTE: When the time of a note is expressed in months, calendar months are used in finding the date of maturity. When the time is expressed in days, the exact number of days is used.

Formerly *three days of grace* were allowed, but since they have been abolished in most states they will not be considered in this book. They should be included in states that still allow them. In that case this note would be due Dec. 8.

All notes due on Saturdays, Sundays, or legal holidays are carried over to the next business day.

4. The **day of discount** is the day when the note is presented to the bank, and they are asked to loan money upon it.

5. The **term of discount** is the number of days from the date when the note is discounted to the date of maturity.

NOTE: In some states the term of discount would include both the day of discount and the day of maturity, making the term one day longer. You should conform to the custom of your locality.

6. If the bank discounts the note on page 100 on Nov. 2, to whom will they look first for payment on Dec. 5?

7. If this note should not be paid by Mr. Jordan on or before Dec. 5, the note is said *to be protested,* and a notary public would immediately notify Mr. Blackmer, and he would be obliged to pay it together with protest fees.

NOTE: A **protest** is a statement made by a notary public certifying that the note has been presented for payment and the payment refused. This notice should be sent to the indorser, or if there is more than one, to each of them within twenty-fours hours after the maker's refusal to honor his note.

Some place of payment is usually indicated on the face of a note. If no place is mentioned, the place of business or the residence of the maker is the proper place to present the note. It is customary also to send to the maker notice of the maturity of a note several days before it is due.

8. Why should a business man be careful not to allow his note to be protested?

1. How much would Mr. Blackmer receive on this note, if he presents it at the bank Nov. 2, 1910?

a. Sept. 5 + 3 mo. = Dec. 5, date of maturity.

b.	*c.*	=		*d.*
Nov. 28	$12.00	= int. 60 da.		$1200.00
Dec. 5	6.00	= int. 30 da.		6.60
33 da.	.60	= int. 3 da.		$1193.40
Term of discount.	$6.60	= int. 33 da.		Proceeds.
	Bank discount.			

To find the proceeds:

a. Find the date of maturity by adding the time specified in the note to the date of the note.

b. Find the term of discount by finding the number of days from the day of discount to the day of maturity.

c. Find the simple interest on the face of the note for the term of discount at the given rate.

d. Find the proceeds by subtracting the discount from the face of the note.

e. If the note is an interest-bearing note, first find the amount of the note at maturity and in c. find the interest upon this amount.

Find the proceeds of the following notes:

	Face	Date	Time	Day of Discount	Rate	Rate of Interest
2.	$260	March 1	2 mo.	March 22	6%	—
3.	$364	April 4	60 da.	May 1	6%	—
4.	$586	June 17	90 da.	July 15	6%	5%
5.	$697	July 20	60 da.	Aug. 11	6%	6%
6.	$450	June 16	3 mo.	June 16	6%	—
7.	$600	Sept. 13	4 mo.	Oct. 13	6%	4%
8.	$796	March 20	75 da.	March 30	6%	—
9.	$500	Feb. 8	90 da.	March 13	6%	5%
10.	$382	Aug. 20	4 mo.	Oct. 13	6%	4%
11.	$496	Sept. 18	2 mo.	Sept. 30	6%	—
12.	$518	Oct. 16	30 da.	Oct. 16	6%	5%

1. You learned on page 98 that banks may issue demand notes, payable to bearer, called **national bank bills**, which circulate as money.

NOTE: Before a bank can issue bills of its own, it must deposit with the Treasury Department in Washington Government Bonds equal in amount to the bills issued. The bills are printed by the government, and the bonds are held in trust for the security of the bill holders.

2. Examine carefully some bank notes and see how they read.

3. Banks are examined at least twice a year by *bank examiners*. This is done as a protection to those who deposit their money in banks.

4. B bought some land for \$2500, and sold it immediately for \$3000 taking in exchange a six months' note without interest. If he had the note discounted at the bank immediately at 6%, how much did he make?

5. Find the proceeds of a note of \$1440, dated Oct. 14, payable in 90 days, and discounted Nov. 21, at 6%.

6. A merchant invested \$1800 in flour, and paid 5% for cartage and other expenses. He sold the total at 20% advance on the total cost, receiving in payment a note for 60 days without interest. If he immediately discounted the note at 6%, what was his entire gain?

7. Write a note for \$60 in favor of your teacher, payable on the First National Bank in 60 days. Have it endorsed to the order of John W. Gould.

8. A 2 months' note for \$850 was dated March 5 and discounted March 23, at 6%. Find the proceeds.

9. F. S. Hall bought of W. M. Cushing 640 bbl. of flour at \$6 a barrel. Write a check for the amount on the National Tradesmens Bank.

10. Find the total proceeds on Sept. 10, 1911, when discounted at 6%, of a note of \$600, dated Aug. 5, 1911, due in 3 mo. without interest, and of a note of \$800, dated Aug. 20, 1911, due in 4 mo. with interest at 4%.

1. A promissory note is a written promise made by one person to pay another a definite sum of money on demand or at a definite time.

2. Study the two forms of notes on pages 30 and 115, till you can write either form from memory.

3. In each of these notes name:

a. The face, the sum of money specified.

b. The place and date when written.

c. The maker, the person who promises to pay.

d. The payee, the person to whom the money is to be paid.

e. The time and place of payment, if mentioned.

f. The rate of interest if an interest bearing note.

g. The indorser, the person who writes his name on the back.

4. When a person makes a payment in part or in full of any bill or account he should receive a receipt. A receipt is a written statement of the receipt of money or other property, and should give the date, the place, the persons interested, and sometimes for what the money was given.

5. The following is a usual blank form to fill out when full payment is made. When only part payment is made the words, "to apply on account" are written in place of "in full of all demands to date."

RECEIPT IN FULL.

$\$$_____ *Boston,*_____ *19*___

*Received of*_____

_____ *Dollars,*

in full of all demands to date

_____

1. A man took a 3 mo. note for \$220 in payment for a horse. He immediately discounted the note at a bank at 6%, and found that he had lost $33\frac{1}{3}\%$ of what the horse originally cost him. Find the cost.

2. A father placed \$100 in a savings bank in his boy's name, when the boy was 10 yr. old. Every half-year the bank added 2% to the deposit for interest, and the father added \$100 every year. What will be the amount on deposit when the boy is 15 years old?

3. If the rate of discount at a bank is 6%, what will be the proceeds of a 3-mo. note of \$570, dated May 23, and discounted June 14?

4. The face of a note is \$2160, the date July 12, the time 3 mo., the date of discount Sept. 9. Find the proceeds.

5. A man borrowed money at 6% and bought 1876 bu. of wheat at 75¢ a bushel, Sept. 5, 1910. On June 15, 1911, he sold all the wheat at $87\frac{1}{2}$¢ a bushel. After paying back the money he had borrowed, how much had he left as his gain?

6. On Sept. 14, 1910, James Murray wrote a check on the Merchants Bank in favor of Chas. S. Holmes for \$150. Write the check.

7. The following day Mr. Holmes sent Mr. Murray a receipt on account. Write the receipt.

8. Mr. C. J. Morse owned 60 acres of woodland. Nov. 15 he sold $\frac{1}{3}$ of the land to R. S. Miller at \$75 an acre and took a 3-mo. note in payment. Write the note.

9. Mr. Morse had the note discounted Dec. 1, at 6%. How much did he receive for the land?

10. State in order the three steps to be taken in finding the bank discount on any note.

11. If a man places \$600 in the savings bank, when the rate of interest is 4% per annum, payable semi-annually, how much can he withdraw at the end of 1 yr. 6 mo.?

1. What is the square of 5? 7^2 = ——. 8^2 = ——.

2. 30^2 = ——. 50^2 =.——. 60^2 = ——.

3. If 40 bu. of corn cost \$12, what will 400 bu. cost?

4. $9 : x = 12 : 4$. Find the value of x.

5. $2 : 8 = 4 : 16$. Read these ratios as fractions.

Change these ratios to fractional forms in their lowest terms:

6. 3 : 15	7. 6 : 12	8. 4 : 16	9. 2 : 10
10. 7 : 3	11. 12 : 5	12. 9 : 2	13. 13 : 4
14. 9 : 3	15. 15 : 3	16. 20 : 5	17. 21 : 7

18. 6 in. is what part of 2 ft.? What is the ratio of 2 ft. to 6 in.?

19. Find the bank discount on a note of \$1200, when the term of discount is 60 days and the rate 6%.

20. A note of \$1000 was discounted for 3 mo. at 6%. Find the proceeds.

21. A 3-mo. note was dated March 10. Find the date of maturity.

22. Who is the payee, in every note or check?

23. The amount of taxable property is \$40,000, and the rate of taxation is 2%. What can you find in this example? Find it.

24. Apple trees are listed in the catalogue at \$1. A farmer bought 100 for \$75. What per cent of discount from the list price did he receive on each tree?

25. The list price of the manufacturer of chairs was \$24 a dozen. A retailer sold them at \$3 each and gained 100%. What rate of discount did he receive from the list price?

26. Ingrain carpet is 1 yd. and Brussels carpet is 27 in. wide. How many yards of each kind of carpet will be needed for a room 9 ft. by 18 ft., if the breadths run lengthwise?

27. What numbers between 24 and 84 are perfect squares?

1. A certain number and $\frac{2}{3}$ of the number equals 15. What is the number?

2. A 60 days' note for \$429 was dated Feb. 21, and discounted Mar. 11, at 5%. Find the proceeds.

3. If a man places \$600 in a savings bank when the rate of interest is 4% per annum, how much can he withdraw at the end of 1 yr. 6 mo., if interest has been compounded semi-annually?

4. How deep must a bin 12 ft. square be made to hold 864 bushels of potatoes?

5. When it is 6.30 A.M. at A, longitude 20° 15′ 20″ E, what time is it at B, 15° 10′ 20″ W?

6. At \$1.87$\frac{1}{2}$ a yard, what will it cost to carpet a room 18 ft. long, 15 ft. wide, with carpet 27 in. wide, if the breadths run lengthwise?

7. A furniture dealer had 800 chairs which cost him \$6 each. A fire destroyed 30% of them, and he sold the remainder at \$8.50 each. How much did he lose?

8. At 2$\frac{1}{2}$% premium a man paid \$177.50 for insurance on his property. What was the value of the property?

9. An agent sold 75 bales of cotton, each bale weighing 350 lb., at 12$\frac{1}{2}$¢ a pound, on a commission of 2$\frac{3}{4}$%. What was his commission? What sum must he return to his employer as net proceeds?

10. Two of the boundary lines of a field run north and south and are 60 rd. and 48 rd. in length. If the distance between these lines is 36 rd., how many acres are there in the field?

11. How many bricks should you estimate by mason's measurement for the 12 in. walls of a house 40 ft. long, 28 ft. wide, and 22 ft. high, if 56 cu. ft. are deducted for openings?

12. At \$6.50 a cord, a pile of 4-ft. wood, 32 ft. long, cost \$35$\frac{3}{4}$. How high was the pile?

1. What dimensions of a square prism do you need to know to find its volume?

2. When you know the volume and one side of the square base, how do you find the altitude of a square prism?

3. One man has loaned a certain sum of money at simple interest at 4%. Another man has placed the same sum of money in the savings bank at 4%. At the end of the 4 years which man will receive the larger amount of interest? Why?

4. State the steps necessary to take in finding the proceeds of a non-interest bearing promissory note.

5. How do you find the square of a number? How do you find the cube of a number?

6. If you know the side of a square, how do you find its area?

7. If you know the side of a cube, how do you find its volume?

8. State two ways of finding out whether four numbers are in proportion.

9. If you know all the terms of a proportion except the last, how can you find that?

10. If you know all the terms of a proportion except the antecedent of the first ratio, how can you find that?

11. State the two ways you have learned for finding the difference in dates. Which method is usually used in examples of interest? Why?

12. Give the name of the third place to the right of the decimal point. Give the last three letters of the word.

13. Give the term in fractions that tells into how many parts the unit has been divided.

14. Give the name of a number that does not refer to any particular thing.

15. If you know the difference in longitude between two places, how do you find the difference in time?

In a school lunch room the following orders for food were prepared:

No. of Orders	Food	Cost	Selling Price	Gain
68	Tomato Soup	$2.00	5¢ per order	
68	Hamburg Steak	2.17	7¢ "	
45	Baked Beans	1.89	6¢ "	
60	Brown Bread	.50	4¢ "	
25	Cocoa	.45	5¢ "	
60	Cottage Pudding	2.89	8¢ "	
90	Salmon Salad	1.35	5¢ "	
48	Gingerbread	1.37	5¢ "	

1. Find the total gain.

2. Find to the nearest hundredth the gain per cent.

3. The school authorities decided that the gain was too great and reduced the price of each order 1 cent. Find the gain on the same number of orders after the reduction was made.

4. Find to the nearest hundredth the gain per cent.

5. On the following day another reduction of 1 cent an order was made. Find the total gain then and the gain per cent.

From your geographies find:

6. The length of North America in degrees.

7. The width in degrees of the United States at its widest part.

8. The number of degrees in length and width of Africa.

9. Ship A is at longitude 5° 30′ 20″ west, and ship B. is at longitude 2° 15′ 10″ east. What is the difference of their longitudes? What is their difference in time?

10. What is the relative position in reference to each other? Which ship has the earlier time?

11. The tax levy in a city is $1,987,275, and the rate is 15 mills on the dollar. What is the assessed valuation?

1. Two towns are 150 miles apart. If the railway fare is \$3.00, what is the rate a mile?

2. Find the cost of $1\frac{1}{2}$ bu. of potatoes at 15¢ a half-peck.

Give results:

3. $8100 \div 100$	4. $66{,}000 \div 22{,}000$	5. 13×300
6. $4634 \div 100$	7. $39{,}000 \div 13{,}000$	8. 2.4×100
9. $40.25 \div 100$	10. $56{,}000 \div 12{,}000$	11. $.12 \times 200$

12. At 32¢ a pound, find the cost of 2 lb. 5 oz. of butter.

13. If 3 men can do a piece of work in 4 days, how long will it take 24 men to do it?

14. What is the effect on the value of a decimal of moving the decimal point two places to the right?

15. If the denominator of a fraction is divided by 3, what is the effect upon the value of the fraction?

16. If a cipher is added at the right of a decimal what effect has it on the value of the decimal?

17. Reduce $\frac{15}{2}$ to a decimal fraction.

18. How many board feet are there in 12 planks, each 14′ long, 6″ wide, and 3″ thick?

19. A room is $\frac{3}{5}$ as wide as it is long, and its length is 20 ft. Find the square feet in the floor.

20. How many days are there from May 18 to July 4?

21. What is the perimeter of a triangle whose sides are $2\frac{1}{2}$ ft., $4\frac{3}{4}$ ft., and $3\frac{1}{4}$ ft.?

22. It requires 64 rd. of fence to inclose a square field. How long is each side of the field?

23. In a school of 300 pupils, 15 are absent. Find the per cent of attendance.

24. How many $\frac{1}{2}$-inch cubes will build a 2-inch cube?

25. A father is $12x$ years old. A son is $\frac{1}{2}$ as old as his father, and a daughter is $\frac{1}{2}$ as old as the son. How old is the daughter?

1. A man traveled 320 miles in three days. If he traveled 3 times as far the first day as he did the third and 4 times as far the second day as the third, how far did he go each day?

2. A bin is 11 ft. 6 in. long, 4 ft. wide, and 6 ft. deep. How many bushels of wheat will it hold? How many of apples?

3. What tax must a man pay if his property is valued at \$5600 in a town where a tax of \$3240 is to be raised? There are 398 polls, each paying \$1.50, and the taxable property is valued at \$440,500.

4. Find the difference between the simple and compound interest of \$660 for 3 yr. 4 mo. 24 d. at 6%.

5. How many board feet are there in 100 boards, each 9 ft. long and 8 in. wide?

6. An employer received from his agent \$1185 as the net proceeds of a sale of \$1250. Find the rate of the agent's commission.

7. Which is better for the purchaser, and how much, a discount of 20% and 10%, or a discount of 30% from a bill of \$3650?

8. A man bought 120 bbl. of apples and sold them for \$345, thereby gaining 15%. What did the apples cost a barrel? If the apples had been sold for \$275, what would have been the loss per cent?

9. Add: 13, 33, 98, 84, 62, 38, 85, 76, 47, 61, 87, 27, 75, 53, 35, 28, 32, 26, 96, 68, 77, 66, 83, 74, 78, 91, 73, 89, 46, 46.

10. If $\frac{1}{7}$ of a number exceeds $\frac{1}{8}$ of it by 224, what is the number?

11. From $\frac{1}{2}$ of a tenth take $\frac{1}{5}$ of a thousandth.

12. A man's crop of wheat brought him \$1687.50, the selling price being 75¢ a bushel. The field in which the wheat grew contained 45 acres. Find the average yield an acre.

1. The sum of two numbers is 24, and one of them is twice as large as the other. What are the numbers?

2. Find the interest on \$1248 from March 4, 1910, to Aug. 12, 1913, at 5%.

3. The area of the gable end of a house is 35 sq. yd. If the house is 42 ft. wide, find the height of the gable.

4. A schoolhouse is insured for \$27,000, which is $\frac{3}{4}$ of its value. If the premium was \$594, at what rate was the house insured? What was the value of the house?

5. The valuation of a town is \$175,600. There are 276 polls, each assessed \$1. The town wishes to raise \$2910. What tax must a man pay who owns a house valued at \$3000?

6. Find the number of gallons of water in a well $4\frac{1}{2}$ ft. in diameter, if the water is 9 ft. deep.

7. A man bought a horse Oct. 3, 1910, for \$186, and gave in payment his note at 6%. On June 13, 1911, he sold the horse for \$195, and paid his note in full. How much did he gain?

8. John Jones paid Amos White of Trenton, N. J., on Sept. 1, 1910, \$112.24, amount due for meat. Write the receipt in full.

9. A gentleman bequeathed $\frac{1}{3}$ of his estate to his wife, $\frac{1}{4}$ of the remainder to his eldest son, and $\frac{1}{5}$ of what then remained to his two daughters, each of whom received \$342. What was the value of the estate?

10. Change 616,346 seconds to minutes, hours, and days.

Find the approximate cost mentally, then test your work:

11. 15 lb. @ 15¢
12. $13\frac{3}{4}$ lb. @ 16¢
13. $28\frac{3}{4}$ cd. @ \$8.
14. $22\frac{1}{4}$ A. @ \$30.
15. $24\frac{1}{2}$ bu. @ 75¢
16. $14\frac{1}{2}$ gal. @ 28¢
17. $18\frac{1}{2}$ doz. @ 24¢
18. 88 yd. @ 15¢
19. 98 cd. @ \$4$\frac{1}{2}$.
20. 110 T. @ \$6.50
21. 76 yd. @ \$1.25
22. 25 pk. @ 25¢.

1. Without changing to cubic feet, find the number of cords of wood in a pile 16 ft. long, 4 ft. wide, and 8 ft. high. In a pile 24 ft. long, 4 ft. wide, and 4 ft. high.

2. A room is 9 ft. square and 10 ft. high. What will it cost to plaster the walls at 25¢ a square yard?

3. Divide the number 45 into two parts, which shall be to each other as 7 to 2.

4. Two men hired a pasture for $40. One pastured 5 horses and the other 3 horses. What ought each to pay?

5. Two boys have 41 marbles. If one has 5 marbles more than the other, how many marbles has each?

6. What will it cost to insure a house for $4200 at $2\frac{1}{2}\%$?

7. What number must be subtracted from $\frac{1}{4}$ to leave .15?

8. A, B, and C, form a partnership: A puts in $3000; B, $2000; and C, $4000; How should a gain of $3600 be divided among them?

9. Find the interest on $3000 for 20 days at 6%.

10. $\frac{1}{4}$ of a number exceeds $\frac{1}{5}$ of it by 9. What is the number?

11. An article was sold for $56 at a loss of 20%. Find the cost and the loss.

12. If a man owns $6000, what tax must he pay, if the rate is $1\frac{1}{2}\%$?

13. If a man insures his life for $4000 at $2\frac{1}{2}\%$ per annum, what will be his annual premium?

14. A man paid $10.50 for having his house insured at $\frac{5}{8}\%$. For what amount was the house insured?

15. If you bought a bill of goods amounting to $25, with a trade discount of 20% and 5% off for cash, what would be the net amount of the bill?

16. When you know the list price of some goods and the two rates of discount in a series, how do you find the net price?

1. Using a scale of $\frac{1}{8}$ in. to a rod, draw a plan of a farm, whose boundary runs as follows: From A 30 rd. east to B; from B, 12 rd. south to C; from C, 24 rd. east to D; from D, 36 rd. south to E; from E, 54 rd. west to F; from F, north to A. Find the whole area of the farm and the length of the boundary fence.

2. A car load of peaches was bought at 80¢ a basket and sold at a loss of $12\frac{1}{2}\%$. If the loss was \$43.20, how many baskets were in the car?

3. After taking out his commission of 4%, an agent remitted to his employer \$1900.80, the amount due him on wheat sold at \$.60 a bushel. How many bushels of wheat were sold?

4. The cellar of a house is 42 ft. long, 33 ft. wide, and 8 ft. deep. Find the cost of digging it at $37\frac{1}{2}$¢ a cubic yard. If the wall is $1\frac{1}{2}$ ft. thick, how many cubic feet are there in the wall? Find the answer both by exact measurement and by mason's measurement.

5. A circular field is 60 rd. in diameter. How many acres does it contain?

6. How many yards of carpeting $\frac{3}{4}$ yd. wide will carpet a room $18\frac{2}{3}$ ft. long and $16\frac{1}{3}$ ft. wide, if the breadths run lengthwise and there is a loss of 7 in. on each breadth for matching?

7. How many bushels of wheat will fill a bin 6 ft. long, $3\frac{1}{2}$ ft. wide, and 3 ft. 8 in. deep?

8. The amount of tax to be assessed in a certain city is \$14,382. The taxable property is \$2,850,800 and the number of polls, each assessed \$1.50, is 1080. What is the rate of taxation?

9. Divide \$4.08 among 4 boys in the proportion of 3, 5, 7, and 9. How many cents will each boy receive?

10. Find the inner and outer circumference of a walk $6\frac{1}{2}$ ft. wide, running round a circular grass plot that measures 90 ft. in diameter. Find the area of the walk.

SECTION II

LESSON 115. *DEFINITIONS*

1. **Arithmetic** is the science that treats of numbers and the art of using them.

2. A **unit** is a single thing or single quantity, used as a standard for measuring other quantities, as one foot, one ounce, one pencil.

3. A **number** is a unit or a collection of units.

4. Number answers the question, *How many*, or shows how great any quantity is when compared with a unit of measurement.

5. An **abstract number** is a number that does not refer to any particular object or unit of measurement. Thus, seven, 36, two hundred ten are abstract numbers.

6. A **concrete number** is a number that does refer to some particular object or unit of measurement. Thus \$3, five books, 7 dozen are concrete numbers.

7. 9 yd.; 4 oz.; 11; \$76; 24; 43; 5 books.

State which of these numbers are concrete and which are abstract.

8. 100 rods; 20 doz. eggs; 116; \$110; 40 minutes.

In each of these numbers tell what is the unit of measurement.

9. When have you had to use your knowledge of arithmetic?

1. **Notation** is the art of expressing numbers by symbols or characters.

2. **Numeration** is the reading of numbers written in symbols or figures.

3. We use the **Arabic System of Notation**, so called because it was first introduced into Europe by the Arabs.

4. This system employs ten characters, called figures, to represent numbers, thus, 0, 1, 2, 3, 4, 5, 6, 7, 8, 9.

5. The first of these characters, 0, is called **cipher**, **zero**, or **nought**. It has no value, and, as we shall see, is used to fill vacant places.

6. The other figures are sometimes called **digits.**

7. A figure standing alone represents *units of the first order*, or simply *units*. The figure at the left of units represents *tens of the unit*, or *units of the second order*. The next figure at the left is *hundreds*, or *units of the third order.*

8. Ten units of any order make one unit of the next higher order, and the units increase in value from right to left.

9. This system of grouping by tens is called a **decimal system**, from the Latin word *decem*, meaning *ten.*

10. For convenience in reading and writing numbers, three orders form a period. The right-hand figure of each group gives the name to the period, as units' period, thousands' period, etc.

11. The periods are separated from each other by a comma, called the **separatrix.**

12. You will rarely ever use numbers above millions. The names of the periods are given here in order from right to left for reference: *units*, *thousands*, *millions*, *billions*, *trillions*, *quadrillions*, *quintillions*, *sextillions*, *septillions*, *octillions*, *nonillions*, *decillions*, etc.

13. Name the periods in order from units to trillions.

14. What name is given to the second period at the left of the decimal point?

Trillions	Hundred-billions	Ten-billions	Billions	,	Hundred-millions	Ten-millions	Millions	,	Hundred-thousands	Ten-thousands	Thousands	,	Hundreds	Tens	Units	Decimal Point	Tenths	Hundredths	Thousandths	Ten-thousandths	Hundred-thousandths	Millionths
5,	2	0	7	,	5	0	3	,	6	9	8	,	0	1	1	.	0	0	0	4	2	6

The number above is read, 5 trillion, 207 billion, 503 million, 698 thousand, 11 *and* 426 millionths.

2. *In reading numbers observe four rules:*

a. Read each period as if it stood alone, and then add the name of the period.

b. Do not read the zero. It merely indicates that that order is omitted.

c. Use the word *and* only in place of the decimal point.

d. Read the decimal as if it were an integer, then add the name of the last or right-hand figure.

3. Notice the similarity of sound in reading:

206,000, two hundred six thousand.
.206, two hundred six thousandths.
200.006, two hundred *and* six thousandths.

4. Notice that the value of any figure in a number depends on two things: —

a. The *absolute* value of the figure without regard to its position; thus, 7 always expresses seven.

b. The *place* or *local* value, which it requires from its position.

5. Thus in the number 777, the first 7 at the right expresses 7 units, the next 7 expresses 7 tens, and the third 7 expresses 7 hundreds.

1. Name the places in order from units to millions, and from units to millionths.

2. What change is made in the value of a figure by moving it one place to the left? What change is made by moving it one place to the right?

Write in figures:

3. Five thousand, four and two tenths.

4. Eleven thousand, two hundred, and three hundredths.

5. One hundred and one thousandth.

6. One hundred one thousandths.

7. Twenty-eight million, two thousand eighty-five, and sixteen millionths.

8. Two hundred trillion, three hundred six million, forty-two, and five hundred-thousandths.

9. One hundred twenty-five thousand, ten, and sixty-seven thousandths.

10. Six billion, eight hundred seventy million, twenty-eight thousand, two hundred six, and one hundredth.

11. One billion, twenty million, three hundred four thousand, fifty, and one hundred five thousandths.

12. Five million, three, and five millionths.

13. Six million, six thousand, six hundred six, and six hundredths.

14. One thousand six and twelve thousand four hundred-millionths.

15. Forty-seven thousand and twelve thousand four millionths.

Read the following numbers:

16. 406.00608
17. 4,060,001,604.00004
18. 1010.6487
19. 28,014,643,005,004.672
20. 100001.01
21. 58,076,104,074.01016
22. 200,200,002.002
23. 5,007,000.004648

1. In the Roman system of notation seven letters are used to express numbers:

Letters	I	V	X	L	C	D	M
Value	1	5	10	50	100	500	1000

2. All other numbers are expressed by combining two or more of these letters.

3. *The following principles must be observed:*

a. When a letter is followed by another letter of equal or less value, the number expressed is equal to the *sum* of the values, thus, III = 3, XV = 15, CC = 200, LXX = 70.

b. When a letter is followed by a letter of greater value than itself, the number expressed is equal to the difference of their values, thus, XC = 90, CD = 400, IX = 9.

c. A bar over a letter multiplies its value by 1000, thus, $\overline{V}$ = 5000, $\overline{XIV}$ = 14,000 $\overline{C}$LX = 100,060.

Express in Arabic notation:

4. XLV	5. CCCV	6. MMM
7. DXC	8. MMD	9. LXV
10. XLVI	11. XCIV	12. LXXXIV
13. CIX	14. DLIV	15. MDCI
16. MXCV	17. MDCCCXIX	18. XCII
19. CMVI	20. MCDX	21. CCXC
22. $\overline{C}$DCCXLIX	23. DCXC	24. DCCX
25. CLXXV	26. XXIX	27. CCCCXLVIII

Express in Roman notation:

28. 16	29. 24	30. 33	31. 52	32. 65
33. 78	34. 89	35. 91	36. 84	37. 47
38. 156	39. 204	40. 560	41. 478	42. 892
43. 999	44. 312	45. 463	46. 421	47. 786
48. 1186	49. 1776	50. 1890	51. 1896	52. 1900

1. **Addition** is the process of uniting two or more numbers into one number.

2. The numbers to be united are called **addends.**

3. The **sum** or **amount** is the result obtained by adding.

4. The sign of addition is +, and is read **plus.**

5. The sign of equality is =, and is read **equals.**

6. **Like numbers** are numbers of the same kind, as, 4 yd. and 7 yd.; \$2 and \$3; 5 and 6.

7. **Unlike numbers** are numbers that are not of the same kind, as, 3 lb. and 4 ft.; \$6 and 5 in.

8. There are three principles to be memorized:

a. Only like numbers can be added.

b. The sum is like the addends.

c. The sum is the same in whatever order the numbers are added.

ORAL EXERCISES

Give sums:

9.	70 80	**10.**	50 60	**11.**	90 40	**12.**	80 30	**13.**	70 90
14.	56 80	**15.**	78 90	**16.**	42 30	**17.**	95 40	**18.**	73 60
19.	28 47	**20.**	86 42	**21.**	57 75	**22.**	68 47	**23.**	96 55
24.	500 480	**25.**	760 300	**26.**	874 600	**27.**	710 873	**28.**	647 560

	33.	**34.**	**35.**	**36.**	**37.**	**38.**	**39.**	**40.**	
29.	7 +	8 +	4 +	9 +	5 +	12 +	8 +	19	=
30.	10 +	9 +	15 +	7 +	13 +	8 +	11 +	7	=
31.	5 +	18 +	7 +	13 +	8 +	9 +	5 +	4	=
32.	8 +	0 +	9 +	7 +	6 +	11 +	7 +	14	=

Add: 657; 724; 452; 817; 865; 108.

657
724
452
817
865
108
33
19
34
3623

For convenience write the addends so that units of the same order are in the same column. Beginning with units' column, add, saying 13, 20, 22, 26, 33. Write it as shown, with the right-hand figure under the column added. Add the other columns and write the results in the same way. By adding the footings of the different columns, we find the sum to be 3623.

This method of retaining the sum of each column, especially of long columns, is advocated. It enables one to find errors in any column without adding other columns.

Arrange as indicated above and check the result by adding upwards and downwards:

1. 7.32 + 28.397 + 11.016 + 248.318 + .428 + 1.08.

2. 81.078 + 16.004 + 8.74 + 164.8 + 82.063 + 8.165.

3. 427.36 + 61.037 + 44.074 + 8.74 + 236.7 + 85.798.

4. 4.84 + 132.468 + 230.067 + 57.8 + 9.732 + 162.875.

5. 5.4307; 48.6512; 7564.02; 314.056; 864.23; 1026-.0087; 2346.002004; 86.24; 1209.00643; 3109.02041; 3287-.0074; 6.007; 704.0049; 1010.00101; 4.07; 16.00109.

6. Thirty-six and twenty-eight thousandths; twenty and eight hundred five ten-thousandths; forty-one thousand, two hundred eight hundred-thousandths; two million, three thousand, one hundred ten, and three thousand sixty-two ten-thousandths; five tenths; twelve hundredths.

7. Six hundred twenty million, two hundred six thousand, four hundred eight; nine million, three thousand, four; twenty-three million, fifteen thousand, five hundred four; seven million, thirty-two thousand, seventy-five; three million, four thousand, forty-eight.

8. $1906.17; $40.24; $5096.08; $7.48; $7091.86; $307.04; $1.04.

1. **Subtraction** is the process of taking one number from another, or it is the process of finding what part of a given number remains, when a part has been taken away.

2. The **minuend** is the number from which we subtract.

3. The **subtrahend** is the number which is subtracted.

4. The result of an example in subtraction is called the **difference** or **remainder.**

5. The sign of subtraction is $-$. It is called the **minus** sign. Thus $11-7$ is read 11 minus 7, and indicates that 7 is to be subtracted from 11.

6. There are two principles in subtraction to be memorized:

a. Only like numbers can be subtracted.

b. The result is like the minuend.

ORAL EXERCISES

7. From 83 take 26. Think of $83-6=77$. $77-20=57$.

Subtract rapidly:

8.	9.	10.	11.	12.
80	45	72	98	30
42	29	36	59	18

13. In examples 8–12, name the subtrahend and minuend.

14. Make change from \$1.00 for the following purchases: \$.47; \$.28; \$.78; \$.59; \$.86; \$.67; \$.49; \$.60.

15. How much larger is 35 than 17? Is the answer a difference or remainder?

16. A boy had 75¢ and spent 48¢. How much money had he left? Is the answer a difference or remainder?

17. The remainder is 35¢, the minuend is 60¢. What is the subtrahend?

18. The difference or remainder added to the subtrahend will give the ————.

19. Subtract by 2's, by 4's, and by 6's from 51.

20. Subtract by 3's, by 5's, and by 7's from 61.

From 382 take 163.

382
163
219

A. *The taking away method.* We cannot take 3 units from 2 units, so we take 1 ten, equal to 10 units, from the 8 tens, leaving 7 tens. We unite this 10 units with 2 units, making 12 units. 12 units less 3 units are 9 units, which we write in units' column. 7 tens less 6 tens is 1 ten, which we write in tens' column. 3 hundreds less 1 hundred are 2 hundreds, which we write in hundreds' column.

In practice we think 3 from 12 = 9, and write 9. 6 from 7 = 1, and write 1. 1 from 3 = 2, and write 2.

B. *The making change or Austrian method.* Here we think what part must be added to 3 to make 12, and added to 6 to make 7, etc. We say 3 and 9 are 12, and write 9. 6 and 1 are 7, and write 1. 1 and 2 are 3, and write 2.

Proof: Add the remainder to the subtrahend. The sum should equal the minuend.

Subtract:

1. 3,040,506,070 − 1,641,874,923
2. 8,976,325 − 4,274,864
3. 7,461,810 − 3,582,468
4. 30,040,060,070 − 14,807,240,635
5. 46,102,060 − 27,085,046
6. 807,060,050 − 356,098,732

7. To what number must 37,648 be added to make 70,601?

8. How much less than 5,002,020 is 2,374,085?

9. What must be subtracted from 200,100,400 that the remainder may be 98,160,710?

10. What number must be added to 647,583 to make 1,047,102?

11. How much greater is one billion than two hundred thirty-two million, forty-five thousand, seven hundred thirty-five?

12. The sum of two numbers is 61,070, and the greater number is 47,564. What is the smaller number?

13. 11,011 less 7769 equals what?

14. How many less than 4178 is 2163?

15. Find the difference between 1376 and 3420.

1. **Multiplication** is the process of repeating a number as many times as there are units in another number.

2. The **multiplicand** is the number repeated.

3. The **multiplier** shows how many times the multiplicand is to be repeated.

4. The **product** is the result of the process of multiplication.

5. The sign of multiplication is $\times$, and is read **times** or **multiplied by**. $4 \times \$3$ is read 4 times \$3, and $\$3 \times 4$ is read \$3 multiplied by 4. In both cases 4 is the multiplier.

6. The multiplicand and multiplier are sometimes called **factors** (makers).

7. Learn the following principles of multiplication:

a. The multiplier must always be an abstract number.

b. The multiplicand may be either an abstract or a concrete number.

c. The product is always like the multiplicand.

d. The product is numerically the same, whichever factor is regarded as the multiplier.

8. The sixty-four facts of multiplication, as found in the multiplication tables, should be thoroughly memorized.

ORAL EXERCISES

9. \$9 $\times$ 7 = ——. 10. 7 $\times$ 8 bu. = ——.

11. 12 in. $\times$ 8 = ——. 12. 9 $\times$ 11 mo. = ——.

13. To multiply any number by 10, annex a cipher to the multiplicand. To multiply by 100, annex 2 ciphers.

Give products at sight:

14.	26	15.	32	16.	71	17.	92	18.	16
	2		4		8		4		3
19.	86	20.	75	21.	100	22.	79	23.	57
	10		10		48		100		1000

1. Multiply 314 by 8.

314
8
2512

8×4 units = 32 units = 3 tens and 2 units. Write the 2 units in units' place and reserve the 3 tens for tens' place. 8×1 ten = 8 tens and the 3 tens reserved make 11 tens, which equal 1 hundred and 1 ten. Write the 1 ten in the tens' place and reserve the 1 hundred for hundreds' place. 8×3 hundreds are 24 hundreds and 1 hundred are 25 hundreds, which we write in its proper place.

In practice we say 8×4 are 32. Put down the 2 and carry the 3. 8×1 are 8 and 3 are 11. Put down the 1 and carry the 1. 8×3 are 24 and 1 are 25. Put down the 25.

2. Multiply 684 by 476.

A.
684
476
4104 = 6×684
47880 = 70×684
273600 = 400×684
325584

B.
684
476
4104
4788
2736
325584

Since ciphers at the right of the partial products do not affect the result, they are usually omitted as in B. Be careful to put the right-hand figure of each partial product directly under the figure of the multiplier that you are using

3. A. Multiply 5600 by 130.

5600
130
168
56
728000

B. Multiply 306 by 204.

(1)
306
204
1224
000
612
62424

(2)
306
204
1224
612
62424

A. Notice that we omit the ciphers in multiplying and annex to the product as many ciphers as we omitted in both multiplier and multiplicand.

B. There are no tens in the multiplier, hence there is no product by tens, as shown in (1). In practice we use the form shown in (2).

Find the product of:

4. 593×698
5. 785×498
6. 637×643
7. 567×797
8. 876×906
9. 807×708
10. 230×460
11. 900×680
12. 608×750
13. 890×906
14. 506×970
15. 730×109
16. 95763×9780
17. 70509×8070
18. 40690×6085
19. 60087×7008
20. 90060×7050
21. 74007×9090

1. What will 943 barrels of flour cost at \$6 a barrel?

(*a*)
$$\begin{array}{r} \$\ 6 \\ 943 \\ \hline 18 \\ 24 \\ 54 \\ \hline \$5658 \end{array}$$

(*b*)
$$\begin{array}{r} 943 \\ \$6 \\ \hline \$5658 \end{array}$$

Method (*a*) is correct and needs no explanation. Since the product is numerically the same, whichever factor is regarded as the multiplier, you should use the other form for brevity. You should keep clearly in mind that \$6 is the multiplicand in each case.

2. What is the value of 8769 acres of land at \$467 an acre?

3. Light travels 186,000 miles a second. If it requires 498 seconds for light from the sun to reach the earth, how far distant is the sun?

4. If it takes the light from a certain star 87,908 seconds to reach the earth, how many miles is it from the earth to the star?

5. If 81,706,800 lb. of tea and 510,048,890 lb. of coffee were imported into the United States in a certain year, find the value of the tea at 45¢ a pound and of the coffee at 24¢ a pound.

6. What is the value of 3986 bales of cotton, each weighing 538 lb., at 23¢ a pound?

7. What is the difference in the cost of 489 head of cattle at \$45.25 a head, and 2384 head of sheep at \$6.75 a head?

8. The United States government bought on contract 1976 reams of foolscap paper at \$3.37½ a ream, and 1850 reams of letter paper at \$2.12½ a ream. How much was paid for the paper?

9. One year the farmers of this country planted 84,204,096 acres of corn. If the average yield was 25 bu. to the acre, how many bushels of corn were raised that year?

10. At 35¢ a bushel for unshelled corn, find the total value of the corn crop that year.

11. In 3 of the western states 7,773,970 head of cattle were sold one year for beef. Find the total value at \$29 a head.

12. Multiply five thousand two by four hundred one.

1. **Division** is the process of finding how many times one number is contained in another, or it is the process of separating a number into equal parts.

NOTE: At 3¢ each, how many peaches can be bought for 15 cents? 15¢ ÷ 3¢ = 5. This illustrates the first definition, or *division*. I paid 15¢ for 5 peaches. What did each cost? 15¢ ÷ 5 = 3¢. This illustrates the second definition, or as it is sometimes called, **partition**.

2. The **dividend** is the number that is to be divided or separated into parts.

3. The **divisor** is the number by which we divide.

4. The **quotient** is the result obtained by division.

5. The **remainder** is the part of the dividend remaining, when the divisor is not contained an exact number of times.

6. The sign of division is ÷, and is read **divided by.**

7. Division is expressed in three ways:

a. Using the sign, thus, $12 \div 4 = 3$.

b. As a fraction, thus, $\frac{12}{4} = 3$.

c. Working form, thus, $4\overline{)12}$.

8. Learn these principles of division:

a. When both divisor and dividend are concrete, they must be like numbers, and the quotient must be an abstract number.

b. When the divisor is abstract and the dividend concrete, the quotient must be like the dividend.

c. Multiplying or dividing both divisor and dividend by the same number does not change the quotient.

ORAL EXERCISES

9. $\$96 \div 8 =$ 10. $8¢\overline{)56¢} =$ 11. $49 \text{ ft.} \div 7 =$

12. $32 \text{ oz.} \div 4 \text{ oz.} =$ 13. $9\overline{)\$63} =$ 14. $32 \text{ oz.} \div 4 =$

15. The divisor is 12, and the quotient 4 in. What is the dividend?

16. The divisor is 9 ft., and the quotient is 9. What is the dividend?

1. Divide 6048 by 36.

A.		B.	
	168		168
	36)6048		36)6048
100 × 36 =	3600		36
Undivided dividend	2448		244
60 × 36 =	2160		216
Undivided dividend	288		288
8 × 36 =	288		288

A. 36 is not contained in 6 thousands any thousands times. It is contained in 60 hundreds 1 hundreds times. Write the 1 over hundreds' figure of the dividend. 36 times 100 are 3600. Subtract it from the dividend. The remainder is 2448. 36 is contained in 244 tens 6 tens times. Write the 6 over the tens' figure of the dividend. 36 times 60 are 2160. Subtract it from the undivided dividend. The remainder is 288. 36 is contained in 288 units 8 times. Write the 8 over the units' figure in the dividend. 36 times 8 units are 288 units. Subtract it from the undivided dividend. The quotient is 168.

B. In practice we use the shorter form shown in B. Here we say, 36 is contained in 60 once. Write 1 as the first figure of our quotient. 36 from 60 = 24. Bring down the 4. 36 is contained in 244 six times. 6 times 36 = 216. 216 from 244 = 28. Bring down the 8. 8 times 36 = 288. There is no remainder. Our quotient is 168.

NOTE: You must be careful to place the first figure of the quotient over the right-hand figure of the part of the dividend used in finding it.

Proof: To prove the work in division multiply the divisor by the quotient, and to the product add the remainder if there is one. The result should equal the dividend.

Divide:

2. 83,927 by 89
3. 109,604 by 94
4. 79,692 by 916
5. 61,074 by 783
6. 13,746,232 by 386
7. 56,882,034 by 594
8. 16,554,184 by 214
9. 40,297,110 by 709
10. 21,106,071 by 687
11. 28,107,090 by 976
12. 16,264,115 by 382
13. 19,075,108 by 456

1. Divide 25,446,182,662 by 62,593.

2. Divide 26,015,867,442 by 75,407.

3. Divide 4,580,423,662 by 27,418.

4. The dividend is 531,288 and the divisor is 628. What is the quotient?

5. How many times can 326 be subtracted from 1,056,566?

6. By what must 4006 be multiplied to make 356,534?

7. The dividend is 17,590,800, and the quotient is 5136. What is the divisor?

8. The product of two factors is 403,326. If one of the factors is 126, what is the other factor?

9. The multiplier is 245, and the product is 562,520. What is the multiplicand?

10. What number will divide 866,775 exactly 195 times?

11. What must be taken from 232,696 that the remainder may be exactly divisible by 512?

12. What is the nearest number to 632,186 that will contain 321 without a remainder?

13. The sum of 365 equal numbers is 3,113,815. What is each number?

14. 75 street cars carried 60,000 passengers in a day. If each car made 10 round trips a day, find the average number of passengers carried each way on each trip.

15. If it costs $10,473,204 to construct 574 miles of railroad, what is the average cost a mile?

16. A company has 750 men on its payroll, and pays them $702,000 wages in a year. What are the average wages for each month for each man?

17. How many pages are there in a book containing 318,240 words, if each page contains 34 lines, and each line contains 9 words?

18. The area of Texas is 265,780 square miles, and that of Massachusetts is 8315 square miles. The area of Texas is approximately how many times that of Massachusetts?

1. A **decimal fraction**, usually called a **decimal**, is a fraction whose denominator is some power of ten.

2. The denominator is not usually written, but is shown by the position of the decimal point.

3. The names of the first six places at the right of the decimal point may be found in the table on page 3.

4. For reference the other names are given: ten-millionths, hundred-millionths, billionths, ten-billionths, hundred-billionths, trillionths, etc.

5. Write the following decimally: seven tenths; twelve hundredths; one hundred two hundred-thousandths; four hundred one thousandths; four hundred and one thousandths; six and ten hundred-thousandths; four and one half tenths; twenty-two hundred and twenty-two hundredths.

6. Add 2.514; 6.7; 18.1205; 8.24.

2.514
6.7
18.1205
8.24
35.5745

For convenience arrange units of the same order in the same column. This will bring the decimal points under one another. Add as in whole or integral numbers, remembering to place the decimal point in the sum below the points in the addends.

7. Subtract 1.07 from 2.79.

2.79
1.07
1.72

Arrange in the same manner as for addition. Subtract as in integral numbers. Remember to place the point in your result below the points in the minuend and subtrahend.

8. Add 4.615; 7.1; .4539; 7.46819; 8.01.

9. Add .612; 5; 9.2178; 5.00623; .2002.

10. Add eighty-two and three hundred sixteen thousandths; one and two hundredths; four and one hundred two ten-thousandths; six thousandths; fourteen.

Subtract:

11. 6.6 − .49	12. 2.106 − .0004	13. 4.01 − 2.004
14. 3.04 − 1.906	15. 3.05 − 1.075	16. 6.4 − 4.806
17. .04 − .025	18. 400 − .004	19. 100 − .0001

1. How does .5 compare with 5? What change is made in the value of a decimal by moving the decimal point one place to the right?

Learn: To multiply a decimal by 10, *move the decimal point one place to the right.*

2. How would you multiply a decimal by 100? By 1000?

Multiply the following by 10, *by* 100, *and by* 1000 *by moving the decimal point:*

3. .3	4. .27	5. .375	6. .256
7. .75	8. .721	9. .074	10. .055
11. .085	12. .007	13. .0025	14. .008

15. Compare .5 with .05. By what has .5 been divided to produce .05?

Learn: To divide a decimal by 10, *move the decimal point one place to the left.*

16. How many places must we move the point to the left to divide by 100? To divide by 1000? To divide by 10,000?

Divide the following by 10, *by* 100, *and by* 1000 *by moving the decimal point:*

17. .8	18. .305	19. 9.08	20. 73.06
21. .05	22. 1.4	23. 7.06	24. 42.1

25. *a.* Multiply 4.4 by 2. *b.* Divide 4.4 by 2.

$$\begin{array}{r} 4.4 \\ \times\ 2 \\ \hline 8.8 \end{array} \qquad 2\overline{)4.4} \quad (2.2)$$

In multiplying or dividing a decimal by an integer the only direction that need be given is to place the decimal point in the product and the quotient, whenever it is reached in the multiplicand or dividend.

Multiply:	*Divide*
26. 1.04 by 5	27. 6.03 by 3
28. 12.4 by 2	29. .015 by 5
30. .005 by 4	31. 20.12 by 4

Multiply 6.24 by 40.

$$\begin{array}{r} 62.4 \\ 4 \\ \hline 249.6 \end{array}$$

Multiplying by 40 is the same as multiplying by 10 and 4. $6.24 \times 10 = 62.4$. $62.4 \times 4 = 249.6$.

Divide 57.6 by 60.

$$6\overline{)5.76}$$ .96

Dividing by 60 is the same as dividing by 10 and 6. $57.6 \div 10 = 5.76$. $5.76 \div 6 = .96$.

Multiply:	*Divide:*
1. 5.075 by 300	2. 91.68 by 80
3. 3.106 by 2000	4. 7.605 by 90
5. 65.05 by 9000	6. 24.64 by 1100
7. 83.6 by 10,000	8. 100.04 by 990
9. 50.016 by 4500	10. 209.576 by 6800
11. 6.074 by 4700	12. 251 1 by 9000
13. 8.075 by 12,000	14. 316.35 by 600
15. .0086 by 20,000	16. 1693.44 by 3600

Multiply 3.5 by .5

a. $\begin{array}{r} .35 \\ 5 \\ \hline 1.75 \end{array}$ *b.* $\begin{array}{r} 3.5 \\ .5 \\ \hline 1.75 \end{array}$

a. This means, find $\frac{5}{10}$ of 3.5. $\frac{1}{10}$ of 3.5 is .35. $\frac{5}{10}$ are $5 \times .35 = 1.75$.

Hence the rule, move the decimal point in the multiplicand as many places to the left as there are decimal places in the multiplier, then multiply as by an integer.

b. In practice we multiply as we multiply in integers, and point off in the product as many decimal places as there are in the multiplicand and multiplier, together.

Multiply:

17. .462, by .005	18. .074 by .641
19. 1.007 by 2.005	20. 1.046 by .0098
21. 5.002 by 5.06	22. 10.005 by .105

23. 15 hundred-thousandths by 76 ten-millionths.
24. 48,062 thousandths by 4078 hundredths.

1. Divide .125 by .05.

$$\begin{array}{r} 2.5 \\ .05\overline{)12.5} \end{array}$$

We have learned that multiplying the divisor and dividend by the same number does not change the quotient. We multiply both terms by 100 by moving the decimal point two places to the right. This makes our divisor an integer, and we divide as by integers.

Learn: Move the decimal point in both divisor and dividend to the right as many places as there are decimal places in the divisor, then divide and point off as in dividing a decimal by an integer.

NOTE: Notice that the number of decimal places in the quotient is equal to the number in the dividend less the number in the divisor.

Divide:

2. 91.512 by 3.72
3. 177.66 by 31.5
4. 151.411 by 6.13
5. 44.591 by 73.1
6. 683.76 by 8.4
7. 20.88 by 8.7
8. 52.26 by 7.8
9. 61.41 by 6.9
10. 872.64 by 86.4
11. .20976 by .46
12. 1.46475 by 31.5
13. 15.01 by 31.6
14. 1.3792 by 8.62
15. 46.224 by 96.3
16. 3.7284 by 4.78
17. .78387 by .087
18. .82848 by .096
19. 199.525 by 57.5
20. 72.1512 by 9.11
21. 67.643 by 1.73

22. The product of two numbers is 35.926, and one of the numbers is 7.81. What is the other number?

23. If a man runs his automobile 59.2 miles a day, in how many days will he run it 1016.084 miles?

24. If 95.25 rods of fence cost $34.29, what is the cost of a rod?

25. How many bushels of corn can be bought for $968.20 at $23\frac{1}{2}$ cents a bushel?

26. If 30.75 yd. of cloth are valued at $199.875, find the value of one yard.

27. Divide 184.23 by 20.7.

1. How much pure iron is there in 64,148 lb. of iron ore, if .75 of it is pure iron?

2. If 40.5 yd. of cloth are bought for $253.125, what will 18.75 yd. cost?

3. If .25 of a farm cost $1200, what will .7 of it cost?

4. Two men bought 2160 acres of western land, and divided it so that one man received .37½ of it, and the other man the remainder. How many acres did each man receive?

5. If 5.625 yd. of carpet cost $7.50, how much will 12.75 yd. cost?

6. Multiply four hundred thousandths by four hundred-thousandths, and divide the product by four tenths.

7. A man sold a mill for $14,500, which was .04 more than he paid for it. How much did he pay for it?

8. How long will it take a man to walk 486 miles, if he walks 3.375 miles an hour and 7.2 hr. each day?

9. Two men divided $16,150.50 between them. One took .625 of it. How much did the other take?

10. Find the cost of 47.625 lb. of meat at 16⅔¢ a pound.

11. A man's real estate is valued at $5004, and that sum is .417 of his personal property. If he divides his entire property among his three children, how much will each child receive?

12. A man sold .375 of his farm for $990, which was at the rate of $52.80 an acre. How many acres were there in the farm at first?

13. How many lengths of fence, each 12.75 ft. long, will it take to inclose a rectangular field that is 522.75 ft. wide, if the length is twice the width?

14. The divisor is .016, and the quotient is 8.62. If there is a remainder of .00015, what is the dividend?

15. How many postage stamps, each containing .72 of a square inch, can be cut from a sheet of 86.4 square inches?

16. Change to decimals and add: $6\frac{3}{4}$, $7\frac{1}{5}$, $3\frac{1}{4}$, $9\frac{1}{2}$, $2\frac{1}{16}$

1. Find the cost of 4.25 cords of wood at $3.75 a cord; 6.5 tons of hay at $12.50 a ton; and 47.75 bu. of potatoes at $.55 a bushel.

2. If a road rises 3.75 ft. in every 50 ft., how much does it rise in a mile? There are 5280 ft. in a mile.

3. A bushel even measure contains 2150.4 cubic inches. If this is .8 of a heaped bushel, how many cubic inches are there in a heaped bushel?

4. A meter is 39.375 inches. How many inches are there in 16 meters? How many meters are there in 17,718.875 inches?

5. If a man travels 29.6 miles a day, in how many days will he travel 1016.168 miles? How many miles will he travel in 7.25 days?

6. How many rods of fence will inclose a rectangular field that is 75.08 rd. long and 46.48 rd. wide?

7. A cubic foot of water weighs 62.5 lb. If pine wood is .4 as heavy as water, how much will 1 cord of pine wood weigh? There are 128 cu. ft. in a cord.

8. If 128.34 yd. of cloth were left in a piece after .08 of the piece had been sold, how many yards were there in the piece at first?

9. If a man can build .375 of a rod of fence in an hour, how many rods can 4 men build in 4.25 days, working 8.5 hours a day?

10. An ounce of pure gold is worth $20.67. How much are 46.25 ounces worth? How many ounces can be bought for $8681.40?

11. A merchant bought 1292 lb. of sugar for $77.52. In order to gain $19.38 for how much a pound must he sell it?

12. Change .0875 to a common fraction in its lowest terms.

13. Find the cost of 36,750 paving blocks at $6.75 a hundred.

14. Multiply 18 thousandths by 15 ten-thousandths and write the product in words.

1. A bushel of oats weighs 32 lb. How many bushels are there in 160 lb.?

2. What is the cost of 5 suits of clothes at $18 each, and of 4 hats at $2.50 each?

3. A boy has 64 stamps in his collection. If he should sell 28 and then buy 16, how many stamps would he have?

4. How much change ought you to receive from 75 cents, after paying 20¢ for some tea and for a pound of coffee, at 35¢?

5. A man bought a parlor set for $60 and a bed-room set for $50. He paid $20 in cash and the balance in 6 equal monthly payments. How much did he pay each month?

6. What is the effect on the value of a decimal, if the decimal point is moved three places to the right?

7. How do you divide an integer by 100? How do you divide a decimal by 1000?

8. What do you mean by minuend? By quotient? By multiplicand?

9. Divide .006 by 100. Multiply the same numbers.

10. How many tenths does it take to make a unit? How many thousandths does it take to make a tenth?

11. In which decimal place should you write millionths?

12. What name should you give to a decimal of eight places?

13. Prove that annexing ciphers to a decimal does not change its value.

14. Name the terms used in division.

15. When both dividend and divisor are concrete, state two principles that you have learned that must follow.

16. Make an example, having the divisor an abstract number and the dividend a concrete number. What name is given to this form of division? What must be true of the dividend and quotient?

17. How much will 30 cows cost at $50 a head?

1. Multiply 15 millionths by 15 thousandths.

2. Divide 8 hundredths by 18 hundredths.

3. At $25.50 an acre, how many acres of land can be bought for $2733.75?

4. At $9\frac{3}{4}$ cents a foot, how many feet of boards can be bought for $220.35?

5. Multiply $62\frac{1}{2}$ thousandths by 25 millionths and divide the product by 125 hundred-thousandths.

6. A man sold 800 bu. of wheat for $864, and lost $16. How much would he have received a bushel, if he had gained $20?

7. A house is worth $3200, and the farm connected with it $800 less than twice as much. The stock and farming tools cost $400 less than the house. What is the value of the whole?

8. What number besides 2832 will divide 1,506,624 without a remainder?

9. The remainder in an example in division is 79. If the quotient is 3 times the remainder, and the divisor is 2 times the quotient, what is the dividend?

10. A farmer in Texas has 1600 sheep. Each sheep yields 6 lb. of wool, which is packed into bales weighing 480 lb. each. He shipped the wool to New York at a cost of $2.88 a bale. If he sold the wool at 36¢ a pound, how much did he receive for it after paying the freight?

11. A farmer in Louisiana had 98 acres of cotton, averaging 235 lb. to the acre. He packed the cotton into bales averaging 490 lb. each. It cost him $1.10 a bale to gin the cotton, and 25¢ a bale for packing. It cost him $2.75 a bale to ship the cotton to Fall River, where he sold it for 7¢ a pound. After paying all expenses, how much did he receive for his cotton crop?

12. A real estate dealer bought 422.25 acres of land. He sold $55\frac{1}{2}$ acres to B, 44.1 acres to C, $83\frac{7}{10}$ acres to D, and the remainder to A. How many acres did he sell to A?

10 mills = 1 cent (¢)	**10 dimes = 1 dollar ($)**
10 cents = 1 dime	**10 dollars = 1 eagle (E)**

The double eagle, eagle, half eagle, and quarter eagle are made of *gold*. The dollar, half dollar, quarter dollar, and dime are made of *silver*. The five-cent piece is made of *nickel*, and the one-cent piece of *bronze*. Paper money in the form of bank notes, treasury notes, gold and silver certificates, is largely used in place of coins. The mill is not coined, but we use the term in speaking of the fractional part of a cent.

Since our United States money system is a decimal system, in which the dollar is the unit, and dimes, cents, and mills are decimal parts of the dollar, it follows that all the rules given for decimals apply to United States money.

ORAL EXERCISES

Read as dollars and cents, also as dollars and decimal parts of a dollar:

1. $10.50	**2.** $4.375	**3.** $75.50	**4.** $1000.15
5. $2.05	**6.** $1.075	**7.** $37.75	**8.** $18,201.10
9. $25.06	**10.** $8.255	**11.** $16.02	**12.** $26,008.04

Change:

13. $15 to cents.

14. 2500 cents to dollars.

15. 17 cents to mills.

16. 125 mills to cents.

17. 20,500 mills to dollars.

18. $19 and 7¢ to cents.

Add:

19. $2.10	**20.** $4.25	**21.** $8.50	**22.** $6.20
1.05	2.50	2.75	5.70
4.04	3.25	1.25	4.80
3.81	2.00	4.50	3.30
23. $8.75	**24.** $7.18	**25.** $1.20	**26.** $2.70
2.50	3.60	2.60	1.15
1.25	2.40	3.20	4.15

1. The receipts from a fair for six consecutive evenings were: \$205.75, \$118.65, \$213.84, \$176.13, \$200.05, and \$318.70. What were the total receipts?

2. At the end of a certain year a merchant had \$7478.15. If his profits for the year were \$2174.89, how much had he at the beginning of the year?

3. A wholesale grocer imported 315 sacks of coffee, each holding 150 lb. If he paid \$.235 a pound, find the value of the coffee.

4. The wages of 130 men for 60 days amounted to \$17,550. How much did each man earn in a day?

5. A dealer sold 112 horses for \$8680, losing \$12.50 on each. What was the cost of each?

6. A real estate dealer paid \$4875 for land at \$75 an acre, and sold a part of it for \$2550 at \$85 an acre. If he sold the remainder at \$72.50 an acre, how much did he gain?

7. A man paid \$655.20 for sheep at \$3.60 each. If the dogs should kill 14, at what price a head must he sell the remainder to suffer no loss?

8. In Dakota a cornfield of 272 acres produced an average yield of corn of 45 bu. an acre. If it cost \$5.35 an acre to raise the corn, and it was sold at 55¢ a bushel, what was the net value from the field?

9. The owner of a 60 acre wheat field in Minnesota kept the following items: Paid for 12 days' plowing at \$2.60, and for 9 days' rolling at \$2.15. Paid for 75 bu. of seed at \$1.25, and for 8 days' work in sowing at \$2.50. Paid for cutting at 80¢ an acre. Paid \$.065 a bushel for threshing the 1400 bu. Sold the wheat at 72¢ a bushel. Find the net profit for the field and the net profit an acre.

10. A woman sold 8 doz. eggs at 30¢ a dozen, and 9 chickens at \$.60 each. With the money she bought 13 bu. of corn. What was the price of a bushel?

11. Find the cost of 386 lb. of tea at $62\frac{1}{2}$¢ a pound.

1. An **account** is a record of business transactions.

2. The **debtor** (**Dr.**) is the person who owes the debt. He is usually the person who bought the goods.

3. The **creditor** (**Cr.**) is the person to whom the debt is owed. He is usually the person of whom the goods were bought.

The following shows the usual form of a ledger account, as shown on the books of the A. H. Fuller Co., in account with Lyman Smith:

Dr.		Lyman Smith					Cr.		
1910					1910				
Sept.	1	20 yd. silk @ $1.00	20	00	Sept.	1	Cash	10	00
"	1	6½ yd. lace @ .36	2	34	"	20	"	5	00
"	15	8 doz. buttons @ .10		80					
"	15	20 yd. flannel @ .30	6	00			Balance	14	14
			29	14				29	14
Oct.	1	Balance due	14	14					

4. What is the sum of the debits? What is the sum of the credits? What is their difference?

5. The difference between the sum of the debits and the sum of the credits is called the **balance.**

6. Notice that the balance is placed on the lesser side and makes the two amounts, called **footings**, equal.

7. Entering the balance and the footings, and ruling the proper lines for a new account is called **closing** or **balancing the account.**

8. On which side of the account are the debit items placed? On which side are the credit items placed?

9. Rule paper and copy the above account without balancing. Add the following items: Oct. 20, bought 6 handkerchiefs @ 25¢, and paid $18. Balance the account. In whose favor is it?

Most business houses present bills to their debtors the first of each month. The following is the bill that the A. H. Fuller Co. would present to Lyman Smith:

New Haven, Oct. 1, 1910.

Lyman Smith,

Bought of the A. H. Fuller Co.

Sept.	1	20 yd. silk	@ $1.00	20	00		
"	1	6½ yd. lace	@ .36	2	34		
"	15	8 doz buttons	@ .10		80		
"	15	20 yd. flannel	@ .30	6	00	29	14
		Cr.					
Sept.	1	By cash		10	00		
"	20			5	00	15	00
		Balance				14	14

1. In this bill who is the debtor? Who is the creditor?

2. Read the first item of the bill. This item should give the *quantity* of the article purchased, its *price*, and its *cost*.

3. State the quantity, price, and cost of each of the other items.

4. Notice carefully where every part of each item is placed.

5. When Mr. Smith pays this bill, the company will write the words "Received Payment," or "Paid," at the bottom and sign their name. This is called **receipting** the bill.

6. When payments are made on an account, a receipt is usually given. The following is the receipt that was given to Mr. Smith, Sept. 1:

New Haven, Sept. 1, '10.

$10.

Received of Lyman Smith, ten dollars to apply on his account.

The A. H. Fuller Co.

1. Many people keep an account of all the money they receive and pay out. This is called a **cash account.**

2. Rule paper the same as for the ledger account in Lesson 140. Write "Cash" in place of Lyman Smith. Put on the debit side all money on hand or money received. Put on the credit side all money paid out.

3. A cash account is balanced the same as the ledger account.

4. How do you find the balance? On which side of the account is it written?

Supply names and dates, write the following as accounts, and find the balance of each:

5. Debits: Painting house, $275; Spraying trees, $36.50; 6 bu. seed potatoes @ $1.50; Garden seeds, $6.25; Ploughing, $10; Fertilizers, $8.00; Feed for hens, $17.50; Cultivator, $15.75.

Credits: Cash on hand, $315.10; 50 bu. potatoes @ 85¢; Vegetables $21.00; 500 doz. eggs @ 28¢; Interest $125.

6. A family account shows the following debits and credits for each month: Credits, Jan. 31, $187.50; Feb. 28, $175.25; Mar. 31, $180.15; April 30, $316.19; May 31, $110.12; June 30, $85.11; July 31, $205.37; Aug. 31, $65.00; Sept. 30, $96.45; Oct. 31, $302.17; Nov. 30, $94.47; Dec. 31, $115.18.

Debits: Jan. 1, $208.19; Feb. 1, $250; Mar. 1, $275; April 1, $390.50; May 1, $190; June 1, $225; July 1, $130; Aug. 1, $25; Sept. 1, $30; Oct. 1, $390.15; Nov. 1, $250.25; Dec. 1, $300.

7. Received $30 on Jan. 1, $25 Feb. 1, $35 March 1, $40 April 1, $55 May 1. Paid out in January $27, in February $30, in March $40, in April $20, in May $38.

8. If you are earning money or have an allowance, keep a cash account for a month.

9. For regular practice ask your parents for permission to keep an account of the household expenses and receipts.

July 5, W. H. Humeston bought of the Central Book Store 3 quires paper @ 20¢; 1 set Dickens Works, $12.50; 3 U. S. Histories @ 65¢. July 18, he bought 15 vols. Irving @ 45¢; 20 doz. pencils @ 30¢. Aug. 5, he bought 3 Geographies @ $1.10 and 6 Arithmetics @ 25¢. Aug. 10, he paid $20 on account.

1. Write this as a ledger account.

2. Write it as a bill to be presented to Mr. Humeston on Sept. 1.

3. Write the receipt that the Central Book Store gave to Mr. Humeston Aug. 10.

4. Supposing Mr. Humeston paid the bill Sept. 10, receipt the bill on that date.

5. The Cash Grocery Store bought of Mata and Torres, Manila, 80 hhd. molasses, 63 gal. each, @ 26¢; 28 sacks sugar, 275 lb. each, @ 4½¢. Make out a receipted bill.

6. Make out and receipt a bill of goods sold to Mary R. Sullivan to-day by the J. R. Smith Co., as follows:

17 yd. cloth @ 30¢; 2 pair shoes @ $3.75; 8 yd. silk @ $1.10.

7. Forbes and Smith imported May 1, 1910, from the Glasgow Mills, Glasgow, Scotland, 6480 yd. of calico @ 4¢, 3790 yd. of gingham @ 12¢, and 2560 yd. of gingham @ 16¢. On June 1, $800 was paid on this account.

a. Write this account as it would appear on the books of the Glasgow Mills.

b. Write a bill to be presented to Forbes & Smith on July 1, 1910. Receipt the bill.

8. A merchant had on hand at the beginning of the year $10,498 worth of goods and $1620 in cash. His sales through the year amounted to $16,475. He paid for goods $14,045, and for clerical expenses $2100. Write as a cash account, and find the balance on hand at the end of the year.

1. What two numbers multiplied together give 21 as their product? Of what two numbers is 15 the product?

2. **Factors** of a number are the integers that, multiplied together, will produce the number.

3. A **factor** then is a number that exactly divides another number.

4. **Factoring** is the process of separating a number into its factors.

5. What are the factors of 9? 18? 27? 35? 49?

6. A **prime number** is a number that has no factors. Although every number can be divided by itself and one, we do not usually call these factors.

7. A **composite number** is a number that has factors.

8. A **prime factor** is a prime number that is a factor. Thus 6 and 3 are factors of 18. Which is a prime factor? Which is a composite factor?

9. Name all the prime numbers between 1 and 25.

10. A number is divisible (can be exactly divided)
   a. By 2, if the units' figure is 0 or is divisible by 2.
   b. By 3, if the sum of the digits is divisible by 3.
   c. By 5, if the units' figure is 0, or is divisible by 5.

11. Applying these tests, name the numbers that are divisible by 2, by 3, or by 5: 14, 21, 45, 111, 270, 3132.

12. Find the prime factors of 105.

$3\overline{)105}$
$5\overline{)35}$
7

Applying the tests in 10, we find that 3 is a factor. Dividing by 3, our quotient is 35. In the same way we find that 5 is a factor. Dividing by 5, our quotient is 7, which is a prime number.

3, 5, 7.

Learn: To factor a number, divide it by any prime factor, and each quotient by any prime factor, until the quotient becomes a prime number. The several divisors and the last quotient are the prime factors.

Find the prime factors of:

1. 429	2. 225	3. 1245	4. 3735
5. 175	6. 342	7. 1050	8. 3224
9. 303	10. 625	11. 1980	12. 5280

GREATEST COMMON FACTOR OR DIVISOR

13. Will 2 and 3 divide both 24 and 36?

14. Because 2 and 3 will exactly divide both 24 and 36 we say that they are *common factors* or *divisors* of 24 and 36.

15. Name all the factors of 24. Name all the factors of 36.

16. Select all the factors that are common to 24 and 36. They are 2, 2, and 3.

17. Since 2, 2, and 3 are the only factors common to 24 and 36, we say that $2 \times 2 \times 3$ or 12 is the greatest common factor or divisor of 24 and 36.

18. The **greatest common divisor** of two or more numbers is the largest number that will exactly divide each of them.

19. Find the greatest common divisor of 45 and 60.

A. $45 = 3 \times 3 \times 5$

$60 = 2 \times 2 \times 3 \times 5$

$3 \times 5 = 15$

B. $3)\underline{45 - 60}$

$5)\underline{15 - 20}$

$3 - 4$

$3 \times 5 = 15$

A. We find that 3, 3, and 5 are the factors of 45, and 2, 2, 3, and 5 are the factors of 60. The only common factors are 3 and 5, therefore 3×5 or 15 is the greatest common divisor.

B. Arrange the numbers as indicated. Divide by any prime factor that will divide both numbers. Divide the quotients by any factor that will divide them both. Continue till there is no factor common to the quotients. The product of the divisors will be the greatest common divisor.

Find the greatest common divisor of:

20. 18, 27, 45.	21. 42, 56, 84.
22. 75, 125, 375.	23. 56, 63, 315.
24. 288, 432, 720.	25. 168, 132, 352.
26. 144, 192, 288.	27. 280, 168, 728.
28. 1136, 3088.	29. 1278, 3474.

1. We say two 2's are 4; three 2's are 6, four 2's are 8, etc. We call these products, 4, 6, and 8, multiples of 2.

2. A **multiple** of any number is the product obtained by multiplying it by some other number.

3. Make a list of the first five multiples of 3.

4. Make a list of the first five multiples of 5.

5. From these lists select a number that is common to both lists. We say then that 15 is the smallest or least multiple that is common to 3 and 5.

6. The **least common multiple** of two or more numbers is the smallest product that is common to them.

7. Since a multiple must be divisible by its factors, it follows that the **least common multiple** of two or more numbers is the least number that is exactly divisible by each of them.

8. Find the least common multiple (abbreviated l. c. m.) of 8, 12, and 20.

A. $8 = 2 \times 2 \times 2$
$12 = 2 \times 2 \times 3$
$20 = 2 \times 2 \times 5$

$2 \times 2 \times 2 \times 3 \times 5 = 120$

B.
$2)8 - 12 - 20$
$2)4 - 6 - 10$
$2)2 - 3 - 5$
$3)1 - 3 - 5$
$5)1 - 1 - 5$
$1 - 1 - 1$

A. Separate each number into its prime factors. The product of these factors, using each factor the greatest number of times it occurs in any one of the numbers, will be the least common multiple.

B. Arrange the numbers as indicated. Divide by any prime number that will divide one or more. When a number is not divisible, bring it down without change. Continue to divide till the quotients are unity. The product of all the divisors will be the least common multiple.

Find the least common multiple of:

9. 28, 42, 63, 108.
10. 171, 592, 703.
11. 65, 78, 104, 130.
12. 115, 165, 230.
13. 45, 63, 72, 99.
14. 105, 210, 330.
15. 90, 110, 165.
16. 75, 90, 260.

Divide 45 by 15.

$$\frac{45}{15} = \frac{\cancel{3} \times 3 \times \cancel{5}}{\cancel{3} \times \cancel{5}} = 3$$

Since $45 = 3 \times 3 \times 5$ and $15 = 3 \times 5$, it follows that $45 \div 15$ is the same as $3 \times 3 \times 5$ divided by 3×5. It is also evident that the factors 3 and 5 may be crossed out or canceled from both dividend and divisor without changing the quotient.

Cancellation is a method of shortening the work in division by rejecting equal factors from dividend and divisor. It is based on the principle that dividing both dividend and divisor by the same number does not change the quotient. (Lesson 127, 8 *c*.)

Divide, using cancellation:

1. $15 \times 28 \times 77$ by $12 \times 24 \times 154$.
2. $9 \times 28 \times 24 \times 40$ by $21 \times 64 \times 45$.
3. $10 \times 15 \times 16$ by 15×48.
4. $24 \times 16 \times 15$ by $48 \times 20 \times 30$.
5. $19 \times 112 \times 15$ by $7 \times 36 \times 50$.
6. $25 \times 14 \times 19$ by $15 \times 21 \times 35$.
7. $70 \times 9 \times 121$ by $30 \times 11 \times 11$.
8. $27 \times 35 \times 42 \times 72$ by $5 \times 9 \times 14 \times 18$.
9. $15 \times 18 \times 48 \times 75$ by $25 \times 30 \times 36 \times 24$.
10. What is the quotient of 60 times the product of 48 and 6, divided by 30 times the product of 24 and 12?
11. A teacher, working 8 months at a salary of \$75 a month, receives the same amount that another teacher receives in 10 months. Find the monthly salary of the second teacher.
12. A farmer bought 42 lb. of sugar at 6¢ a pound and gave in exchange 9 doz. eggs. How much did he receive a dozen for his eggs?
13. How many neckties at 50¢ can you get for a credit check received for returning 10 yd. of gingham at 15¢ a yard?
14. A tree 45 ft. high casts a shadow 75 ft. long. What length of shadow will be cast by a tree 111 ft. high?

1. A **fraction** is one or more of the equal parts of a unit.

2. If we speak of three-fourths of an apple, we must *think* of the apple as divided into 4 parts.

3. We write that expression thus $\frac{3}{4}$.

4. The **denominator names** the parts, that is, it tells us into how many parts the unit is divided.

5. The **numerator numbers** the parts, that is, it tells us *how many* parts are used.

6. The numerator is the number above the line, and the denominator is the number below the line.

7. The numerator and denominator are called the **terms** of the fraction.

8. Name the numerator and denominator in each of these fractions: $\frac{5}{7}$, $\frac{2}{3}$, $\frac{4}{5}$, $\frac{9}{10}$.

9. Fractions are classified with respect to their *value* into proper and improper fractions.

10. A **proper fraction** is one whose numerator is less than its denominator, that is, its value is less than a unit, as $\frac{5}{6}$.

11. An **improper fraction**, or a **fractional number**, is one whose numerator is equal to or greater than its denominator, that is, its value is equal to or greater than a unit, as, $\frac{2}{2}$, $\frac{9}{6}$, $\frac{15}{12}$, etc.

12. Fractions are classified with respect to their *form* into simple and complex fractions.

13. A **simple fraction** is one whose terms are integers, as $\frac{4}{5}$.

14. A **complex fraction** is one which contains a fraction in one or both of its terms, as $\frac{2\frac{1}{2}}{4}$, $\frac{3}{5\frac{1}{6}}$, $\frac{2\frac{1}{2}}{3\frac{3}{4}}$.

15. A **mixed number** is a number consisting of an integer and a fraction, as $4\frac{1}{2}$, $7\frac{1}{9}$.

16. Classify the following fractions according to their value and form: $\frac{3}{4}$, $\frac{7}{8}$, $\frac{9}{7}$.

1. **Reduction** of a fraction is the process of changing its form without changing its value.

2. Every fraction is an expression of division. The numerator is the dividend and the denominator is the divisor.

Learn: Multiplying or dividing both terms of a fraction by the same number does not change its value. See Lesson 127, 8 *c*.

3. Change $\frac{30}{35}$ to its lowest terms.

$\frac{30 \div 5 = 6}{35 \div 5 = 7}$ We divide both terms by the common factor 5, according to the principle stated above.

Learn: To change a fraction to its lowest terms divide both terms by a common factor, till no common factor can be found.

NOTE: When no common factor can be found by inspection, find the greatest common divisor of the terms, and divide both terms by it.

Change to lowest terms:

ORAL EXERCISES

4. $\frac{63}{74}$	**5.** $\frac{36}{48}$	**6.** $\frac{42}{56}$	**7.** $\frac{27}{54}$	**8.** $\frac{44}{55}$
9. $\frac{80}{88}$	**10.** $\frac{25}{45}$	**11.** $\frac{24}{54}$	**12.** $\frac{18}{21}$	**13.** $\frac{14}{28}$
14. $\frac{8}{48}$	**15.** $\frac{9}{36}$	**16.** $\frac{11}{44}$	**17.** $\frac{12}{60}$	**18.** $\frac{21}{49}$
19. $\frac{25}{75}$	**20.** $\frac{16}{24}$	**21.** $\frac{27}{45}$	**22.** $\frac{22}{33}$	**23.** $\frac{48}{60}$

WRITTEN EXERCISES

24. $\frac{336}{126}$	**25.** $\frac{190}{380}$	**26.** $\frac{216}{252}$	**27.** $\frac{144}{432}$	**28.** $\frac{625}{5625}$
29. $\frac{213}{324}$	**30.** $\frac{792}{830}$	**31.** $\frac{231}{308}$	**32.** $\frac{132}{144}$	**33.** $\frac{735}{1260}$
34. $\frac{209}{638}$	**35.** $\frac{415}{913}$	**36.** $\frac{78}{273}$	**37.** $\frac{144}{156}$	**38.** $\frac{11777}{12984}$
39. $\frac{108}{324}$	**40.** $\frac{143}{361}$	**41.** $\frac{272}{546}$	**42.** $\frac{192}{256}$	**43.** $\frac{868}{1092}$
44. $\frac{430}{516}$	**45.** $\frac{234}{312}$	**46.** $\frac{410}{492}$	**47.** $\frac{168}{224}$	**48.** $\frac{858}{1287}$
49. $\frac{324}{396}$	**50.** $\frac{180}{288}$	**51.** $\frac{144}{192}$	**52.** $\frac{114}{171}$	**53.** $\frac{26744}{45584}$

Change $\frac{5}{6}$ to 36ths.

$\frac{5}{6} \times \frac{6}{6} = \frac{30}{36}$ Since multiplying both terms by the same number does not change the value of the fraction, we multiply both terms by 6.

Learn: To change a fraction to higher terms, multiply both terms by such a number as will change the denominator to the required denominator.

NOTE: This multiplier is found by dividing the required denominator by the given denominator.

Change:

1. $\frac{2}{3}$, $\frac{3}{4}$, $\frac{5}{6}$ to 12ths.
2. $\frac{1}{4}$, $\frac{1}{3}$, $\frac{7}{8}$, $\frac{5}{12}$ to 48ths.
3. $\frac{2}{3}$, $\frac{1}{2}$, $\frac{5}{6}$ to 18ths.
4. $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{4}$, $\frac{5}{6}$ to 36ths.
5. $\frac{3}{4}$, $\frac{5}{6}$, $\frac{7}{8}$ to 24ths.
6. $\frac{4}{5}$, $\frac{2}{3}$, $\frac{1}{4}$, $\frac{5}{12}$ to 60ths.
7. Change $19\frac{5}{6}$ to sixths.

$\frac{6}{6} \times 19 = \frac{114}{6}$

$\frac{114}{6} + \frac{5}{6} = \frac{119}{6}$

In 1 unit there are 6 sixths. In 19 units there are 19 times $\frac{6}{6}$ or $\frac{114}{6}$ and $\frac{5}{6}$ are $\frac{119}{6}$.

Learn: To change a mixed number to an improper fraction, change the integer to an equivalent fraction, and then unite the two fractional parts.

$$\begin{array}{r} 19 \\ 6 \\ \hline 114 \end{array}$$

$\frac{114}{6} + \frac{5}{6} = \frac{119}{6}$

In practice we multiply the integer by the denominator and to the product add the numerator, writing this sum over the denominator.

Change to improper fractions:

8. $15\frac{3}{4}$
9. $46\frac{11}{12}$
10. $38\frac{15}{16}$
11. $16\frac{3}{8}$
12. $96\frac{4}{5}$
13. $41\frac{4}{15}$
14. $29\frac{3}{7}$
15. $87\frac{7}{11}$
16. $16\frac{7}{25}$
17. $498\frac{7}{16}$
18. $27\frac{7}{8}$
19. $74\frac{2}{5}$
20. $96\frac{7}{9}$
21. $35\frac{11}{15}$
22. $144\frac{8}{11}$
23. $49\frac{1}{2}$
24. $63\frac{2}{7}$
25. $71\frac{7}{12}$
26. $61\frac{9}{10}$
27. $173\frac{3}{4}$

Change $\frac{27}{5}$ to a mixed number.

$5\overline{)27}$
$5\frac{2}{5}$

Since there are $\frac{5}{5}$ in one unit, in $\frac{27}{5}$ there must be as many units as $\frac{5}{5}$ is contained times in $\frac{27}{5}$ or $5\frac{2}{5}$.

Learn: To change an improper fraction to an integer or a mixed number, divide the numerator by the denominator.

Change to integers or mixed numbers:

1. $\frac{86}{7}$	**2.** $\frac{91}{9}$	**3.** $\frac{87}{13}$	**4.** $\frac{71}{14}$	**5.** $\frac{252}{9}$
6. $\frac{893}{17}$	**7.** $\frac{458}{23}$	**8.** $\frac{583}{37}$	**9.** $\frac{324}{17}$	**10.** $\frac{237}{15}$
11. $\frac{345}{24}$	**12.** $\frac{796}{31}$	**13.** $\frac{288}{72}$	**14.** $\frac{576}{96}$	**15.** $\frac{218}{28}$
16. $\frac{896}{128}$	**17.** $\frac{625}{25}$	**18.** $\frac{672}{28}$	**19.** $\frac{956}{16}$	**20.** $\frac{401}{84}$

Change $\frac{3}{8}$ to a decimal.

$.375$
$8\overline{)3.000}$

$\frac{3}{8}$ means $3 \div 8$. Annex zeros to the numerator and divide by the denominator.

A. $.28\frac{4}{7}$
$7\overline{)2.00}$

B. $.28571 +$
$7\overline{)2.00000}$

Some common fractions cannot be changed to an exact equivalent in the form of a decimal. In such cases we may express the remainder as a common fraction as in A, or we may carry our division to any number of decimal places. The + following indicates incomplete division.

Learn: To change a common fraction to a decimal annex zeros to the numerator and divide by the denominator.

Change to equivalent decimals, carrying none beyond millionths.

21. $\frac{3}{5}$	**22.** $\frac{7}{8}$	**23.** $\frac{3}{16}$	**24.** $\frac{4}{25}$	**25.** $\frac{5}{16}$
26. $6\frac{1}{64}$	**27.** $4\frac{1}{8}$	**28.** $7\frac{9}{25}$	**29.** $3\frac{17}{32}$	**30.** $4\frac{13}{15}$
31. $.4\frac{1}{2}$	**32.** $.7\frac{5}{8}$	**33.** $.9\frac{3}{20}$	**34.** $.2\frac{17}{40}$	**35.** $.1\frac{1}{50}$

1. Change .025 to a fraction.

$.025 = \frac{25}{1000} = \frac{1}{40}$ We write the decimal as a common fraction, and reduce it to its lowest terms.

Change the following decimals to common fractions:

2. .64	**3.** .032	**4.** .12½	**5.** .87½
6. .00125	**7.** .0005	**8.** .024	**9.** .1625
10. .081¼	**11.** .0725	**12.** 2.03⅘	**13.** 75.25
14. .53125	**15.** 7.125	**16.** .008	**17.** 1.875

ADDITION OF FRACTIONS

18. What kind of numbers can be added or subtracted?

19. What change must you make before you can add 2 yd. and 24 in.?

20. What kind of fractions can be added or subtracted?

21. Similar fractions are fractions having the same denominator.

Learn: Fractions must be made similar before they can be added or subtracted.

22. Add $\frac{3}{4}$ and $\frac{2}{5}$.

$\frac{3}{4} = \frac{15}{20}$. $\frac{2}{5} = \frac{8}{20}$.

$\frac{15}{20} + \frac{8}{20} = \frac{23}{20} = 1\frac{3}{20}$.

We make these fractions similar by finding the least common multiple of the denominators, which is 20. Changing both fractions to 20ths and adding the numerators, we have for the sum $\frac{23}{20}$ or $1\frac{3}{20}$.

23. Add $4\frac{2}{9}$ and $6\frac{5}{6}$.

A.
$$\begin{array}{r|l} & 18 \\ \hline 4\frac{2}{9} & 4 \\ 6\frac{5}{6} & 15 \\ \hline 10\ \frac{19}{18} & \end{array} = 1\frac{1}{18}$$
$$\begin{array}{r} 1\frac{1}{18} \\ \hline 11\frac{1}{18} \end{array}$$

B.
$$\begin{array}{rl} 4\frac{2}{9} = & 4\frac{4}{18} \\ 6\frac{5}{6} = & 6\frac{15}{18} \\ \hline & 10\frac{19}{18} = 11\frac{1}{18} \end{array}$$

In addition of mixed numbers, add the integers and fractions separately.

Add:

24. $\frac{19}{24}$, $\frac{13}{36}$, $\frac{2}{3}$, $\frac{3}{4}$, $\frac{1}{2}$.

25. $\frac{3}{8}$, $\frac{5}{6}$, $\frac{2}{9}$, $\frac{7}{12}$, $\frac{3}{4}$.

26. $17\frac{3}{7}$, $11\frac{11}{14}$, $8\frac{1}{2}$.

27. $4\frac{7}{16}$, $9\frac{7}{8}$, $2\frac{3}{4}$, $1\frac{1}{2}$.

1. From $\frac{5}{6}$ take $\frac{3}{8}$.

$\frac{5}{6} = \frac{20}{24}$. $\frac{3}{8} = \frac{9}{24}$. $\frac{20}{24} - \frac{9}{24} = \frac{11}{24}$.

We change both fractions to similar fractions, then find the difference of the numerators, and write it over the common denominator.

2. From $11\frac{1}{2}$ take $7\frac{7}{9}$.

$11\frac{1}{2} = 11\frac{9}{18} = 10\frac{27}{18}$

$7\frac{7}{9} = 7\frac{14}{18} = 7\frac{14}{18}$

$3\frac{13}{18}$

After changing the fractions to similar fractions, we cannot take $\frac{14}{18}$ from $\frac{9}{18}$, therefore we take 1 from 11, change it to 18ths, and unite it with $\frac{9}{18}$, making the minuend $10\frac{27}{18}$. From $10\frac{27}{18}$ we can take $7\frac{14}{18}$.

Add and subtract:

3. $112\frac{5}{6}$ / $61\frac{2}{9}$

4. $176\frac{3}{4}$ / $78\frac{5}{8}$

5. $41\frac{17}{36}$ / $37\frac{5}{7}$

6. $21\frac{8}{15}$ / $10\frac{1}{2}$

7. $21\frac{3}{8}$ / $14\frac{18}{16}$

8. $85\frac{9}{32}$ / $41\frac{1}{2}$

9. $28\frac{7}{15}$ / $15\frac{2}{3}$

10. $96\frac{3}{4}$ / $64\frac{2}{3}$

11. $81\frac{17}{66}$ / $45\frac{7}{9}$

12. $66\frac{9}{24}$ / $41\frac{8}{36}$

13. $45\frac{3}{7}$ / $18\frac{5}{6}$

14. $96\frac{7}{12}$ / $12\frac{4}{5}$

15. $29\frac{1}{5}$ / $11\frac{4}{7}$

16. $74\frac{5}{9}$ / $11\frac{7}{15}$

17. $24\frac{1}{3}$ / $12\frac{1}{2}$

18. $86\frac{3}{4}$ / $27\frac{2}{5}$

19. What number diminished by $17\frac{5}{9}$ leaves $16\frac{7}{15}$?

20. A merchant had three pieces of silk, each containing $30\frac{1}{2}$ yards. From one piece he sold $21\frac{3}{4}$ yd., from another $19\frac{2}{3}$ yd., and from the third piece $24\frac{5}{8}$ yd. How many yards were left in the three pieces?

21. George earned $\$6\frac{2}{5}$ the first week, $\$5\frac{1}{4}$ the second week, $\$8\frac{9}{10}$ the third week, and $\$10\frac{1}{2}$ the fourth week of February. How much did he earn during the month?

22. A lives $11\frac{1}{3}$ miles east of Hartford, and B lives $7\frac{1}{2}$ miles farther east. If C lives $9\frac{3}{4}$ miles west of Hartford, how far apart do A and B live? How far apart do A and C live? B and C?

23. What number must be subtracted from $714\frac{1}{3}$ to leave $416\frac{3}{4}$ as a remainder?

1. Multiply (*a*) $\frac{3}{4}$ by 5. (*b*) $\frac{7}{10}$ by 5.

(*a*) $\frac{3 \times 5}{4} = \frac{15}{4} = 3\frac{3}{4}$. (*b*) $\frac{7 \times \cancel{5}}{\cancel{10}\ 2} = \frac{7}{2} = 3\frac{1}{2}$.

Learn: To multiply a fraction by an integer, multiply the numerator of the fraction by the integer.

NOTE: Arrange the numbers as indicated for multiplication and cancel when possible, as shown in (*b*). This is the same as dividing the denominator of the fraction by the integer. This method should be used when possible.

2. At $\$2\frac{1}{4}$ a yard how much will 7 yd. of cloth cost?

$\$2\frac{1}{4}$
7
$1\frac{3}{4}$
14
$\$15\frac{3}{4}$

If 1 yd. costs $\$2\frac{1}{4}$, 7 yd. will cost 7 times $\$2\frac{1}{4}$. 7 times $\$\frac{1}{4} = \$\frac{7}{4} = \$1\frac{3}{4}$. 7 times \$2 = \$14. $\$14 + \$1\frac{3}{4} = \$15\frac{3}{4}$.

Learn: To multiply a mixed number by an integer, multiply the integer and the fractional parts separately, and add the products.

Multiply:

3. $\frac{5}{6}$ by 8
4. $\frac{9}{10}$ by 5
5. $\frac{7}{12}$ by 8
6. $\frac{11}{15}$ by 9
7. $\frac{11}{16}$ by 124
8. $\frac{11}{12}$ by 180
9. $43\frac{3}{8}$ by 16
10. $36\frac{7}{12}$ by 18
11. $48\frac{9}{32}$ by 16
12. $115\frac{1}{5}$ by 35
13. $209\frac{1}{8}$ by 24
14. $117\frac{1}{2}$ by 64

15. If an express train runs $58\frac{3}{4}$ miles in an hour, how many miles will it run in 10 hours?

16. A boy lives $1\frac{3}{8}$ miles from school. How far must he travel in a week, if he goes home to dinner?

17. If a book costs $\$\frac{4}{5}$, how much will 22 books cost?

18. At the rate of $\frac{7}{8}$ of a mile in a minute, how far does an automobile travel in 45 minutes?

19. Find the cost of 36 lb. of butter at $28\frac{1}{2}¢$ a pound.

20. What is the value of 19 cd. of wood at $\$7\frac{1}{8}$ a cord?

1. (*a*) Multiply 13 by $\frac{2}{3}$. (*b*) Multiply 16 by $\frac{3}{4}$.

$$13 \times \frac{2}{3} = \frac{26}{3} = 8\frac{2}{3} \qquad \overset{4}{\cancel{16}} \times \frac{3}{\cancel{4}} = 12$$

Learn: To multiply an integer by a fraction, multiply the integer by the numerator of the fraction and divide by the denominator.

NOTE: When possible use cancellation as in (*b*).

2. Multiply 95 by $5\frac{3}{4}$.

$$\begin{array}{r} 95 \\ 5\frac{3}{4} \\ \hline 4)285 \\ \frac{3}{4} \times 95 = \quad 71\frac{1}{4} \\ 5 \times 95 = \quad 475 \\ \hline 5\frac{3}{4} \times 95 = \quad 546\frac{1}{4} \end{array}$$

Multiply the integer by the fraction as shown here, when possible. Notice that we multiply the integer by the fractional part of the mixed number and then by the whole part.

Multiply:

3. 75 by $\frac{17}{50}$
4. 625 by $\frac{21}{25}$
5. 27 by $\frac{2}{9}$
6. 32 by $\frac{9}{16}$
7. 22 by $\frac{7}{33}$
8. 24 by $\frac{19}{2}$
9. 54 by $\frac{17}{27}$
10. 132 by $\frac{3}{8}$
11. 994 by $\frac{3}{4}$
12. 42 by $15\frac{2}{3}$
13. 122 by $18\frac{2}{7}$
14. 3342 by $26\frac{2}{3}$
15. 47 by $8\frac{2}{3}$
16. 214 by $24\frac{3}{4}$
17. 1240 by $17\frac{1}{5}$
18. 62 by $11\frac{5}{6}$
19. 364 by $15\frac{2}{3}$
20. 2676 by $95\frac{6}{6}$
21. Find the cost of $12\frac{1}{2}$ doz. lemons at 20¢ a dozen.
22. Find the cost of $56\frac{3}{4}$ gal. of oil at \$.12 a gallon.
23. What is the cost of $15\frac{2}{3}$ cd. of wood at \$6 a cord, and $12\frac{1}{4}$ cd. at \$4 a cord?
24. What is the cost of $\frac{7}{12}$ of an acre of land at \$92 an acre?
25. What is the cost of $\frac{7}{15}$ of a ton of coal at \$7.50 a ton?
26. What is the cost of $42\frac{3}{4}$ lb. of sugar at 7¢ a pound?
27. Hay worth \$18 a ton was sold for $\frac{5}{6}$ of its value. At that price what was received for $5\frac{1}{2}$ tons?

1. Multiply $\frac{5}{6}$ by $\frac{2}{5}$.

A. $\frac{1}{6}$ of $\frac{5}{6} = \frac{1}{6}$.
$\frac{2}{5} = 2 \times \frac{1}{6} = \frac{2}{6} = \frac{1}{3}$.

This means find $\frac{2}{5}$ of $\frac{5}{6}$. We first find $\frac{1}{5}$ of $\frac{5}{6}$, which is $\frac{1}{6}$. $\frac{2}{5}$ then must be 2 times $\frac{1}{6}$, which is $\frac{2}{6}$ or $\frac{1}{3}$.

B. $\frac{\cancel{5}}{\cancel{6}_{3}} \times \frac{2}{\cancel{5}} = \frac{1}{3}$.

The work may be much shortened by canceling all factors common to the numerators and denominators.

Learn: To multiply a fraction by a fraction, multiply the numerators together for the new numerator and the denominators for the new denominator of the product. Always cancel when possible.

2. Multiply $15\frac{2}{3}$ by $12\frac{1}{5}$.

A. $15\frac{2}{3} = \frac{47}{3}$. $12\frac{1}{5} = \frac{61}{5}$.
$\frac{47}{3} \times \frac{61}{5} = \frac{2867}{15} = 191\frac{2}{15}$.

A. We may change the mixed numbers to improper fractions. Then multiply as suggested in example 1.

B. $15\frac{2}{3}$
$12\frac{1}{5}$
$3\frac{2}{15} = \frac{1}{5} \times 15\frac{2}{3}$
$8 = 12 \times \frac{2}{3}$
$180 = 12 \times 15$.
$191\frac{2}{15}$

B. We may multiply without changing to improper fractions. $\frac{1}{5} \times \frac{2}{3} = \frac{2}{15}$. $\frac{1}{5} \times 15 = 3$. $12 \times \frac{2}{3} = 8$. $12 \times 15 = 180$. This is a good method to use when one is accustomed to it, especially when the numbers are small.

3. Multiply $5\frac{1}{2} \times \frac{3}{11} \times \frac{5}{12} \times 1\frac{2}{15}$.

$\frac{\cancel{11}}{2} \times \frac{\cancel{3}}{\cancel{11}} \times \frac{\cancel{5}}{\cancel{12}} \times \frac{\cancel{12}}{\cancel{15}} = \frac{1}{2}$.

The sign $\times$ between these fractions is usually read "of." A fraction of a fraction is called a **compound fraction.** The rule for multiplying is the same as that given above.

Multiply:

4. $\frac{8}{9}$ by $\frac{3}{4}$	**5.** $1\frac{2}{25}$ by $\frac{5}{6}$	**6.** $3\frac{2}{39}$ by $1\frac{8}{16}$
7. $8\frac{1}{82}$ by $2\frac{6}{27}$	**8.** $4\frac{4}{45}$ by $1\frac{5}{17}$	**9.** $2\frac{0}{21}$ by $1\frac{5}{16}$
10. $85\frac{3}{4}$ by $24\frac{1}{5}$	**11.** $25\frac{7}{8}$ by $2\frac{4}{5}$	**12.** $16\frac{2}{3}$ by $6\frac{3}{8}$
13. $19\frac{1}{2}$ by $15\frac{1}{3}$	**14.** $46\frac{1}{3}$ by $36\frac{1}{2}$	**15.** $74\frac{1}{3}$ by $26\frac{4}{5}$
16. $21\frac{1}{5}$ by $35\frac{1}{3}$	**17.** $81\frac{1}{4}$ by $16\frac{2}{3}$	**18.** $56\frac{4}{7}$ by $49\frac{3}{8}$

Multiply:

1. $\frac{11}{25}$ by $\frac{35}{44}$
2. $\frac{27}{64}$ by $\frac{20}{81}$
3. $\frac{16}{75}$ by $\frac{25}{65}$
4. $15\frac{1}{2}$ by $9\frac{2}{3}$
5. $24\frac{1}{3}$ by $12\frac{1}{2}$
6. $27\frac{7}{10}$ by $20\frac{5}{9}$
7. $69\frac{1}{3}$ by $72\frac{5}{8}$
8. $76\frac{1}{2}$ by $7\frac{1}{3}$
9. $86\frac{3}{4}$ by $27\frac{3}{5}$
10. $\frac{19}{28} \times \frac{14}{57} \times \frac{15}{16} \times \frac{32}{45}$
11. $12\frac{2}{3} \times 8\frac{4}{9} \times 4\frac{1}{10} \times \frac{3}{76}$
12. $3\frac{2}{3} \times \frac{51}{70} \times 6\frac{2}{3} \times \frac{21}{25}$
13. $3\frac{1}{3} \times \frac{7}{15} \times 2\frac{1}{7} \times \frac{5}{9}$

14. A merchant bought 60 yd. of cloth at $\$2\frac{1}{4}$ a yard. He sold $\frac{5}{6}$ of it at $\$2\frac{3}{4}$ a yard, and the remainder at $\$2\frac{1}{2}$ a yard. Find his gain.

15. At $\$1\frac{7}{8}$ a rod, find the cost of building $21\frac{1}{4}$ rd. of fence.

16. A cubic foot of water weighs $62\frac{1}{2}$ lb. If copper is $8\frac{4}{5}$ times as heavy as water, what is the weight of a cubic foot of copper?

17. George has $\frac{3}{5}$ of \$425. Samuel has $\frac{2}{3}$ as much as George. If John has $\frac{5}{9}$ as much as both the others, how much has John?

18. A owned $\frac{7}{8}$ of a factory and sold $\frac{4}{5}$ of his share to B. How much did A then own? If B should sell $\frac{2}{3}$ of his share to C, how much would B still own? What part of the factory would C own?

19. A man who owned \$23,475 failed and could pay only $\frac{7}{8}$ of his debts. How much could he pay?

20. If a train travels at the rate of $45\frac{1}{2}$ miles an hour, how far will it travel in $5\frac{1}{3}$ hours?

21. There are $16\frac{1}{2}$ ft. in a rod. How many feet are there in $16\frac{1}{2}$ rods?

22. A man sowed 36 acres of land with oats, and $5\frac{7}{9}$ times as much with wheat. How many acres of oats and wheat were there?

23. Find the product of $504\frac{1}{4}$ and $64\frac{5}{8}$.

24. If you multiply $264\frac{3}{4}$ by $48\frac{5}{8}$, what will be the product?

25. What is the rent for $72\frac{7}{8}$ acres of pasture land at $\$2\frac{3}{4}$ an acre for the season?

1. Divide $\frac{6}{7}$ by 3.

(*a*) $\frac{\overset{2}{\cancel{6}}}{7} \div \cancel{3}_{1} = \frac{2}{7}$.

(*b*) $\frac{\overset{2}{\cancel{6}}}{7} \times \frac{1}{\underset{1}{\cancel{3}}} = \frac{2}{7}$.

This means find $\frac{1}{3}$ of $\frac{6}{7}$. This is done by dividing the numerator of the fraction by the integer as in (*a*), or we may express the work as in multiplication, and use cancellation as in (*b*).

2. Divide $\frac{5}{7}$ by 3.

$\frac{5}{7} \times 3 = \frac{5}{21}$

In this case we cannot divide the numerator, so we multiply the denominator by the integer, since multiplying the denominator divides the fraction.

3. Divide $25\frac{2}{3}$ by 6.

(*a*) $6\overline{)25\frac{2}{3}}$ $\quad 4\frac{5}{18}$

(*a*) Here we divide as in whole numbers. 6 is contained in $25\frac{2}{3}$ four times and $1\frac{2}{3}$ remainder. $1\frac{2}{3} = \frac{5}{3}$, $\frac{5}{3} \div 6 = \frac{5}{18}$.

(*b*) $25\frac{2}{3} = \frac{77}{3}$.

$\frac{77}{3} \times \frac{1}{6} = \frac{77}{18} = 4\frac{5}{18}$.

(*b*) We may change the mixed number to an improper fraction and divide as in example 1.

Learn: To divide a fraction by an integer, divide the numerator or multiply the denominator of the fraction by the integer, canceling when possible.

ORAL AND WRITTEN EXERCISES

Divide:

4. $\frac{6}{7}$ by 2	**5.** $2\frac{1}{2}$ by 5	**6.** $3\frac{1}{5}$ by 8
7. $\frac{2}{3}$ by 7	**8.** $\frac{3}{4}$ by 4	**9.** $\frac{3}{5}$ by 6
10. $4\frac{2}{5}$ by 12	**11.** $3\frac{5}{2}$ by 5	**12.** $3\frac{5}{8}$ by 7
13. $37\frac{1}{2}$ by 15	**14.** $62\frac{1}{2}$ by 25	**15.** $25\frac{5}{7}$ by 5
16. $264\frac{4}{5}$ by 4	**17.** $341\frac{1}{3}$ by 3	**18.** $216\frac{5}{8}$ by 6
19. $726\frac{1}{2}$ by 24	**20.** $631\frac{4}{5}$ by 36	**21.** $581\frac{1}{3}$ by 32
22. $3218\frac{7}{8}$ by 42	**23.** $3249\frac{2}{3}$ by 65	**24.** $6242\frac{1}{2}$ by 54
25. $1602\frac{1}{2}$ by 45	**26.** $7631\frac{1}{2}$ by 24	**27.** $1275\frac{3}{8}$ by 56
28. $1030\frac{2}{3}$ by 16	**29.** $1691\frac{1}{2}$ by 32	**30.** $2346\frac{1}{12}$ by 84
31. $1311\frac{3}{7}$ by 36	**32.** $2876\frac{3}{4}$ by 24	**33.** $1875\frac{2}{3}$ by 81

1. Divide 6 by $\frac{2}{3}$.

$6 = \frac{18}{3}$.
$\frac{18}{3} \div \frac{2}{3} = 18 \div 2 = 9$.

We change the integer to a fraction having the same denominator as the fraction, and then divide the numerators.

Learn: To divide an integer by a fraction, change the integer to a fraction whose denominator is the same as that of the divisor and then divide the numerator of the dividend by the numerator of the divisor.

2. Divide $\frac{2}{3}$ by $\frac{3}{4}$.

A. $\frac{2}{3} = \frac{8}{12}$. $\frac{3}{4} = \frac{9}{12}$.
$\frac{8}{12} \div \frac{9}{12} = 8 \div 9 = \frac{8}{9}$.

In this example the common denominator is 12, so we change both fractions to 12ths, and then divide the numerators.

B. $\frac{2}{3} \div \frac{3}{4} = \frac{2}{3} \times \frac{4}{3} = \frac{8}{9}$.

Since 1 divided by $\frac{3}{4}$ is $\frac{4}{3}$, it follows that $\frac{2}{3} \div \frac{3}{4}$ would be $\frac{2}{3}$ of $\frac{4}{3}$, or $\frac{8}{9}$. This is the same as multiplying by the divisor inverted.

Learn: A. *To divide a fraction by a fraction, change both fractions to fractions having a common denominator and then divide the numerator of the dividend by the numerator of the divisor.*

B. *Multiply the dividend by the inverted divisor.*

NOTE: Before dividing, change mixed numbers to improper fractions.

ORAL AND WRITTEN EXERCISES

Divide:

3. 4 by $\frac{5}{6}$
4. 8 by $\frac{2}{5}$
5. 3 by $\frac{3}{4}$
6. $\frac{3}{4}$ by $\frac{1}{2}$
7. $\frac{2}{3}$ by $\frac{1}{6}$
8. $\frac{5}{6}$ by $\frac{2}{3}$
9. 16 by $\frac{8}{9}$
10. 74 by $7\frac{2}{5}$
11. 60 by $1\frac{5}{7}$
12. $\frac{5}{8}$ by $\frac{3}{7}$
13. $\frac{3}{5}$ by $\frac{1}{2}$
14. $\frac{3}{8}$ by $5\frac{1}{4}$
15. $4\frac{1}{3}$ by $2\frac{1}{2}$
16. $8\frac{1}{3}$ by $1\frac{2}{5}$
17. $4\frac{1}{2}$ by $2\frac{1}{3}$
18. $69\frac{1}{4}$ by $4\frac{1}{8}$
19. $42\frac{1}{4}$ by $2\frac{5}{8}$
20. $12\frac{1}{2}$ by $1\frac{3}{4}$
21. $64\frac{3}{5}$ by $1\frac{3}{4}$
22. $116\frac{3}{8}$ by $36\frac{3}{4}$
23. $16\frac{1}{5}$ by $3\frac{3}{8}$
24. $166\frac{2}{3}$ by $12\frac{1}{2}$
25. $163\frac{21}{25}$ by $12\frac{4}{5}$
26. $481\frac{1}{4}$ by $137\frac{1}{2}$
27. $157\frac{1}{2}$ by $17\frac{1}{2}$
28. $126\frac{2}{5}$ by $15\frac{4}{5}$
29. $250\frac{1}{6}$ by $26\frac{1}{3}$

1. A **complex fraction** is one having a fraction for one or both of its terms.

2. A complex fraction is **simplified** by performing the operations indicated.

3. Simplify: $\dfrac{2\frac{1}{6} + 1\frac{1}{4}}{4\frac{1}{2} - 3\frac{1}{8}}$

$2\frac{1}{6} + 1\frac{1}{4} = 3\frac{5}{12} = \frac{41}{12}$.
$4\frac{1}{2} - 3\frac{1}{8} = 1\frac{3}{8} = \frac{11}{8}$.
$\frac{41}{12} \div \frac{11}{8} = \frac{82}{24} \div \frac{33}{24} = 2\frac{16}{33}$.

We first add $2\frac{1}{6}$ and $1\frac{1}{4}$. The result is $\frac{41}{12}$. Next we subtract $3\frac{1}{8}$ from $4\frac{1}{2}$ and obtain $\frac{11}{8}$. $\frac{41}{12} \div \frac{11}{8} = 2\frac{16}{33}$.

Simplify:

4. $\dfrac{3\frac{1}{4}}{4\frac{1}{2}}$

5. $\dfrac{6\frac{2}{5}}{8\frac{4}{5}}$

6. $\dfrac{7\frac{5}{8}}{4\frac{3}{8}}$

7. $\dfrac{40\frac{5}{6}}{6\frac{2}{3}}$

8. $\dfrac{\frac{2}{5} \times 13\frac{1}{4}}{\frac{4}{7} \times 7\frac{7}{8}}$

9. $\dfrac{\frac{3}{4} \times 9\frac{2}{3}}{9\frac{2}{3} \div \frac{7}{8}}$

10. $\dfrac{\frac{1}{2} + \frac{2}{3}}{\frac{3}{4} + \frac{5}{6}}$

11. $\dfrac{1\frac{5}{6} + 2\frac{1}{2}}{2\frac{1}{4} - 1\frac{1}{3}}$

12. A man has a yearly income of \$4200. If he spends $\frac{1}{6}$ of it the first year, $\frac{1}{7}$ of it the second, and $\frac{1}{3}$ of it the third year, how much can he save in the three years?

13. If $1\frac{1}{3}$ acres of land cost \$76, what will $17\frac{3}{19}$ acres cost?

14. If $18\frac{3}{4}$ tons of iron cost \$285, how much will $10\frac{9}{10}$ tons cost?

15. If a man walks $3\frac{3}{4}$ miles an hour, how many hours will it take him to walk $187\frac{1}{2}$ miles?

16. If $\frac{7}{8}$ of a yard of satin costs \$2.10, how much will $49\frac{1}{2}$ yd. cost?

17. If the multiplicand is $7\frac{1}{2}$, and the product $3\frac{1}{3}$, what is the multiplier?

18. A man paid $\$33\frac{3}{4}$ for cloth at $\$3\frac{3}{5}$ a yard. How many yards did he buy?

19. The product of three numbers is $124\frac{2}{3}$. Two of the numbers are $7\frac{1}{5}$ and $8\frac{1}{4}$. What is the third?

20. A lot containing $19\frac{1}{5}$ acres was divided into 32 equal parts. What part of an acre was in each part?

1. If $\frac{3}{4}$ of a yard of cloth costs 21¢, what does 1 yard cost?

A. Since $\frac{3}{4}$ yd. costs 21¢,

$\frac{1}{4}$ yd. will cost $\frac{1}{3}$ of 21¢ or 7¢.

$\frac{4}{4}$ yd. or 1 yd. will cost 4 times 7¢ = 28¢.

B. $21¢ \times \frac{4}{3} = 28¢$.

Method A shows the manner of solving this problem by analysis. A study of the solution shows us that we obtain the same result by multiplying by the *reciprocal* of the fraction, that is, by the fraction inverted.

2. If $\frac{3}{4}$ of a yard of cloth costs 21¢, what will $3\frac{1}{2}$ yd. cost?

$21¢ \times \frac{4}{3} \times \frac{7}{2} =$ ———.

Since 1 yd. of cloth costs $21¢ \times \frac{4}{3}$, it follows that $3\frac{1}{2}$ yd. will cost $3\frac{1}{2}$ times that sum, or $21¢ \times \frac{4}{3} \times \frac{7}{2}$.

NOTE: In problem work of this kind have the pupils make statements of all the processes to be used, and then shorten the work by cancellation.

3. If $\frac{5}{8}$ of an acre of land costs \$75, what will $2\frac{1}{3}$ acres cost?

4. If a man raises $4\frac{1}{2}$ tons of hay on $2\frac{3}{4}$ acres of land, how many tons can he raise on $12\frac{3}{8}$ acres?

5. If $5\frac{1}{2}$ bu. of wheat cost \$6.60, how many bushels can be bought for $\$121\frac{1}{5}$?

6. If a man can build $\frac{1}{1}\frac{2}{7}$ of a wall in $5\frac{1}{3}$ days, how long will it take him to finish it?

7. A has $\$12\frac{1}{2}$ and B $\$6\frac{2}{5}$. If C has $\frac{7}{9}$ as much as A and B together, how much money has C?

8. A man owned $\frac{4}{5}$ of a farm and sold $\frac{1}{3}$ of his share. If what he had left was worth \$224, what was the value of the whole farm?

9. \$45 is $\frac{1}{4}$ more than what sum of money? This means, \$45 is $\frac{5}{4}$ of what sum of money?

10. \$24 is $\frac{1}{3}$ less than what sum of money? This means, \$24 is $\frac{2}{3}$ of what sum of money?

11. \$27 is $\frac{1}{2}$ more than what sum of money?

12. \$81 is $\frac{1}{3}$ less than what sum of money?

13. If $\frac{3}{4}$ of a yard of cloth is worth 75¢, find the cost of $17\frac{1}{3}$ yards. Find how many yards can be bought for $\$76\frac{1}{4}$.

1. 4 is what part of 12?

$\frac{4}{12} = \frac{1}{3}$. Since 1 is $\frac{1}{12}$ of 12, it follows that 4 must be 4 times $\frac{1}{12}$ or $\frac{4}{12} = \frac{1}{3}$ of 12.

2. $\frac{3}{5}$ is what part of $\frac{2}{3}$?

$\frac{3}{5} = \frac{9}{15}$. $\frac{2}{3} = \frac{10}{15}$.

$9 \div 10 = \frac{9}{10}$.

or

$\frac{3}{5} \div \frac{2}{3} = \frac{3}{5} \times \frac{3}{2} = \frac{9}{10}$.

Since $\frac{3}{5} = \frac{9}{15}$ and $\frac{2}{3} = \frac{10}{15}$, it follows that $\frac{3}{5}$ is the same part of $\frac{2}{3}$ that 9 is of 10, that is, $\frac{9}{10}$.

We obtain the same result by dividing the first fraction by the second.

Learn: To find what part one number is of another, write the first number as the numerator of a fraction, the second number as its denominator, and reduce the fraction to its lowest terms.

ORAL EXERCISES

Find what part:

3. 5 is of 15. **4.** 4 is of 16. **5.** 2 is of 12.

6. $\frac{1}{4}$ is of $\frac{1}{2}$. **7.** $\frac{1}{8}$ is of $\frac{1}{4}$. **8.** $\frac{1}{6}$ is of $\frac{1}{3}$.

9. $2\frac{1}{2}$ is of 5. **10.** $3\frac{1}{3}$ is of 10. **11.** $3\frac{1}{2}$ is of 7.

12. $12\frac{1}{2}$ is of 100. **13.** $16\frac{2}{3}$ is of 100. **14.** $33\frac{1}{3}$ is of 100.

Find what part of a \$1.00 *is:*

15. 10¢ **16.** 20¢ **17.** 25¢ **18.** 40¢

19. 50¢ **20.** 60¢ **21.** 75¢ **22.** 80¢

23. $33\frac{1}{3}$¢ **24.** $12\frac{1}{2}$¢ **25.** $66\frac{2}{3}$¢ **26.** $37\frac{1}{2}$¢

27. $62\frac{1}{2}$¢ **28.** $87\frac{1}{2}$¢ **29.** \$1.25 **30.** \$1.50

31. \$1.20 **32.** \$1.$33\frac{1}{3}$ **33.** \$1.60 **34.** \$1.$62\frac{1}{2}$

35. There are 135 pupils in the eighth grade. If 50 girls and 40 boys enter the high school, what part of the class will enter?

36. $3\frac{1}{2}$ miles is what part of $10\frac{1}{2}$ miles?

37. 1 qt. 1 pt. is what part of one gallon?

38. 24 oz. is what part of 3 pounds?

39. \$$4\frac{1}{2}$ is what part of \$$22\frac{1}{2}$?

40. $\frac{3}{8}$ is what part of $1\frac{1}{2}$?

1. If $5\frac{1}{2}$ tons of coal are worth $\$35\frac{3}{4}$, how much are $25\frac{1}{4}$ tons worth? How many tons can you buy for $\$178\frac{3}{4}$?

2. If $1\frac{5}{6}$ baskets of peaches cost \$1.76, what will $7\frac{1}{4}$ baskets cost?

3. If $3\frac{1}{2}$ bbl. of flour are worth \$21, what are $21\frac{1}{2}$ bbl. worth? How many barrels can be bought for \$507?

4. What will a man pay for his board for a year or 52 weeks at the rate of $\$47\frac{1}{2}$ for $9\frac{1}{2}$ weeks?

5. A grocer sold $4\frac{1}{2}$ lb. of butter for \$1.44. At the same price how much would he receive for $28\frac{1}{4}$ lb.?

6. When $\frac{7}{8}$ of a barrel of flour costs \$4.20, what will 64 barrels cost?

7. If $8\frac{1}{2}$ tons of hay cost \$136, how many tons can be bought for \$1156?

8. If 6 barrels of apples cost $\$9\frac{3}{4}$, how many barrels can be bought for \$184?

9. If $1\frac{1}{3}$ acres of land cost \$100, what will $79\frac{1}{2}$ acres cost?

10. If $5\frac{1}{2}$ cords of wood cost $\$24\frac{3}{4}$, what will $17\frac{1}{2}$ cords cost?

11. If $\frac{3}{4}$ of a gallon of molasses costs 36¢, what is the price of $26\frac{1}{2}$ gallons?

12. When butter is worth 16 cents a half pound, how much must you pay for $15\frac{1}{4}$ pounds?

13. If $\frac{3}{5}$ of a farm is worth \$7350, what is $\frac{2}{3}$ of it worth?

14. If 8 baskets of peaches are worth $\$12\frac{4}{5}$, how many baskets can be bought for $\$353\frac{3}{5}$?

15. If $\frac{9}{10}$ of a ton of coal costs \$5.67, what must a man pay for $213\frac{3}{4}$ tons?

16. If a railroad conductor travels 1400 miles in $17\frac{1}{2}$ days, how far will he travel in $87\frac{1}{2}$ days?

17. If a man earns $\$5\frac{2}{5}$ in 3 days, how many days must he work to earn $\$219\frac{3}{5}$?

18. If $\frac{3}{4}$ of a ton of hay is worth \$15.90, how much are 36.75 tons worth at the same price?

1. What is the cost of $5\frac{1}{8}$ lb. of cheese at 24¢ a pound?

2. 32 is how many times 16? If 16 sheep cost $96, what will 32 sheep cost?

3. What change is made in the value of a fraction by multiplying its numerator and denominator by 4? Why?

4. Divide .256 by 100. Multiply the same numbers.

5. In every fraction what does the denominator show? What does the numerator show?

6. What is a factor? Give 2 factors of 15.

7. What do you mean by factoring? Factor 15.

8. What will $\frac{5}{9}$ of a yard of cloth cost, if 5 yards cost 90 cents?

9. How many strawberry plants can you put in a row $15\frac{1}{2}$ ft. long, if the plants are half a foot apart?

10. If $\frac{7}{8}$ of a yard of silk is worth 56 cents, what will $2\frac{1}{2}$ yd. be worth?

11. What is the least common multiple of 12, 15, 20, and 60?

12. What is the greatest common divisor of 9, 27, and 45?

13. If 8 yd. of muslin cost 64 cents, what is the cost of 12 yards?

14. A load of wheat was sold for $19.50 at 65¢ a bushel. How many bushels were there in the load?

15. Of what number is 5 one of the three equal factors?

16. If it takes 12 men 18 days to do a piece of work, how long will it take 6 men? 24 men? 4 men?

17. How much will 240 bales of cotton weigh at 300 pounds a bale?

18. A dealer sold 20 automobiles at $1200 each. How much did he receive for them all?

19. Name the two kinds of notation.

20. What is meant by the place value of a figure? Illustrate your meaning by the number 222.

1. The population of New York is 4,766,883; of Chicago, 2,185,283; of Philadelphia, 1,549,008; of St. Louis, 687,029; of Boston, 670,585. Find the population of these five largest cities in the Unites States.

2. The area of New York City is 327 sq. mi.; of Chicago, 179.5 sq. mi.; of Philadelphia, 127.8 sq. mi.; of St. Louis, 61.3 sq. mi.; of Boston, 38.4 sq. mi. Find to the nearest hundredth the number of people to each square mile in each city.

3. Find the density of population in a city whose population is 553,669 and whose area is 30.1 sq. mi.

4. California exports 2000 car loads of celery valued at $800,000. Find the average value of a car load.

5. California values the walnuts exported last year at $1,716,000. If 8000 tons were exported, what is the price of a ton of walnuts?

6. It is estimated that 1 orange tree will produce 5 boxes of oranges, and an average of 80 trees are planted on an acre. If there is a net gain to the grower of $1 a box, find his net gain from 10 acres of land.

7. The total value of the products of California in 1908 was $405,115,431, and the area of the state is 153,650 square miles. What was the value of the products for each square mile?

8. The population of the state the same year was 2,564,363. If the value of the products had been divided equally among the entire population, how much would each person have received?

9. An agent sold two farms for $4000 each. On one he gained $\frac{1}{5}$ of the cost, and on the other he lost $\frac{1}{5}$. Did he gain or lose and how much?

10. Find the sum of $26\frac{1}{15}$, $37\frac{9}{10}$, $16\frac{1}{6}$, $48\frac{3}{5}$.

11. From 120 acres of land $44\frac{1}{5}$ acres were sold to one man, and $\frac{1}{3}$ of the remainder to another. Find the value at $75 an acre of the part not sold.

1. How many times will \$1 contain 50 cents? \$.25? \$.20? $33\frac{1}{3}$ cents?

2. The equal parts of any number are called its **aliquot parts.**

3. There are some aliquot parts of \$1, and some multiples of these parts that occur so frequently in business that you should be familiar with them.

If you are not already familiar with the following, you should commit them to memory now:

$\frac{1}{2} = .50$	$\frac{1}{8} = .12\frac{1}{2}$	$\frac{3}{5} = .60$	$\frac{7}{8} = .87\frac{1}{2}$
$\frac{1}{4} = .25$	$\frac{1}{10} = .10$	$\frac{4}{5} = .80$	$1\frac{1}{2} = 1.50$
$\frac{1}{3} = .33\frac{1}{3}$	$\frac{3}{4} = .75$	$\frac{5}{6} = .83\frac{1}{3}$	$1\frac{1}{4} = 1.25$
$\frac{1}{5} = .20$	$\frac{2}{3} = .66\frac{2}{3}$	$\frac{3}{8} = .37\frac{1}{2}$	$1\frac{1}{3} = 1.33\frac{1}{3}$
$\frac{1}{6} = .16\frac{2}{3}$	$\frac{2}{5} = .40$	$\frac{5}{8} = .62\frac{1}{2}$	$1\frac{1}{8} = 1.12\frac{1}{2}$

4. Find the cost of 36 yd. of ribbon at \$.$33\frac{1}{3}$ a yard.

Since \$.$33\frac{1}{3}$ = \$$\frac{1}{3}$, 1 *yd. costs* \$$\frac{1}{3}$, and 36 *yd. cost* $36 \times \$\frac{1}{3}$ = \$12.

ORAL EXERCISES

Find the cost of:

5. 27 yd. of ribbon at \$.$33\frac{1}{3}$.
6. 80 cakes of soap at \$.25.
7. 44 lb. of meat at \$.25.
8. 40 bu. potatoes at \$.50.
9. 66 lb. butter at \$.$33\frac{1}{3}$.
10. 36 pk. apples at \$.$12\frac{1}{2}$.
11. 33 lb. coffee at \$.$33\frac{1}{3}$.
12. 32 lb. tea at \$.50.
13. 20 lb. fish at \$.10.
14. 10 gal. oil at \$.20.
15. 40 doz. eggs at \$.25.
16. 35 yd. cloth at \$.20.
17. 60 lb. sugar at \$.05.
18. 72 collars at \$.$16\frac{2}{3}$.
19. 30 grape fruit at 5¢.
20. 2000 ft. boards at 10¢.
21. 48 cans corn at 20¢.
22. 10 bars soap at $6\frac{1}{4}$¢.
23. 48 articles at $12\frac{1}{2}$¢.
24. 64 articles at $12\frac{1}{2}$¢.
25. 48 articles at $16\frac{2}{3}$¢.
26. 64 articles at 25¢.
27. 100 articles at 10¢.
28. 64 articles at 50¢.
29. 48 articles at $33\frac{1}{3}$¢.
30. 75 articles at $33\frac{1}{3}$¢.

1. Find the cost of 90 rd. fence at $\$.83\frac{1}{3}$ a rod.

A. $\cancel{90}\ 15 \times \frac{\$5}{\cancel{6}} = \$75.$

A. Since $\$.83\frac{1}{3} = \$\frac{5}{6}$, 1 rd. will cost $\$\frac{5}{6}$. 90 rd. will cost 90 times $\$\frac{5}{6} = \$75$.

B. $\$90$ = cost at $\$1$
$\$15$ = cost at $\$\frac{1}{6}$
$\$75$ = cost at $\$\frac{5}{6}$

B. $\$90$ = cost at $\$1$ a rod. Since the price is only $\$\frac{5}{6}$, we must deduct $\frac{1}{6}$ of the cost, or $\$15$. The remainder is $\$75$.

2. Find the cost of 96 yd. of cloth at $\$1.25$ a yard.

A. $\cancel{96}\ 24 \times \frac{\$5}{\cancel{4}} = \$120.$

A. Since $\$1.25 = \$\frac{5}{4}$, 1 yd. will cost $\$\frac{5}{4}$. 96 yd. will cost 96 times $\$\frac{5}{4}$, or $\$120$.

B. $\$96$ = cost at $\$1$
$\$24$ = cost at $25¢$
$\$120$ = cost at $\$1.25$

B. $\$96$ = cost at $\$1$. $\frac{1}{4}$ of $\$96 = \$24$, cost at $25¢$. $\$96 + \$24 = \$120$.

3. Find the cost of 64 chairs at $\$12.50$.

$\$6400$ = cost at $\$100$
$\$800$ = cost at $\$12.50$

$\$12.50$ is $\frac{1}{8}$ of $\$100$. First find the cost at $\$100$ by annexing two zeros. It follows that the cost at $\$12.50$ will be $\frac{1}{8}$ of $\$6400$, or $\$800$.

4. Find the cost of 24 acres of land at $\$250$ an acre.

$\$24000$ = cost at $\$1000$.
$\$6000$ = cost at $\$250$.

First find the cost at $\$1000$, and then find $\frac{1}{4}$ of that sum, since $\$250$ is $\frac{1}{4}$ of $\$1000$.

Find the cost of:

5. 72 bu. corn at $87\frac{1}{2}¢$.
6. 72 horses at $\$87\frac{1}{2}$.
7. 992 yd. cloth at $12\frac{1}{2}¢$.
8. 672 chairs at $\$12\frac{1}{2}$.
9. 92 lb. coffee at $25¢$.
10. 68 books at $\$2.50$.
11. 135 yd. cloth at $66\frac{2}{3}¢$.
12. 78 cows at $\$66\frac{2}{3}$.
13. 136 articles at $62\frac{1}{2}¢$.
14. 280 articles at $\$62\frac{1}{2}$.
15. 84 articles at $75¢$.
16. 148 articles at $\$75$.
17. 714 gal. molasses at $66\frac{2}{3}¢$.
18. 416 lb. tea at $66\frac{2}{3}¢$.
19. 124 bu. potatoes at $87\frac{1}{2}¢$.
20. 70 horses at $\$87\frac{1}{2}$.
21. 36 pairs skates at $\$1.87\frac{1}{2}$.
22. 90 yd. cloth at $\$2.16\frac{2}{3}$.
23. 56 books at $\$1.75$.
24. 320 yd. silk at $75¢$.

1. At $33\frac{1}{3}¢$ a yard, how many yards of cloth can be bought for \$45?

45 yd.
3
135 yd.

Since at $33\frac{1}{3}¢$ a yard you can buy 3 yd. for \$1, it follows that for \$45 you can buy 45 times 3 yd., or 135 yd.; or, since at \$1 a yard you can buy 45 yd. for \$45, then at $33\frac{1}{3}¢$ you can buy 3 times 45 yd., since \$1 is 3 times $33\frac{1}{3}¢$.

2. At $\$.37\frac{1}{2}$ a pound, how many pounds of coffee can you buy for \$18?

$18 \text{ lb.} \times \frac{8}{3} = 48 \text{ lb.}$

At \$1 a pound you can buy 18 lb. for \$18. Since \$1 is $\frac{8}{3}$ times $\$.37\frac{1}{2}$, it follows that at $37\frac{1}{2}¢$ a pound you can buy $\frac{8}{3}$ times 18 lb., or 48 lb.

3. At \$1.25 a pair, how many pairs of shoes can a dealer buy for \$75?

$75 \text{ pairs} \times \frac{4}{5} = 60 \text{ pairs.}$

At \$1 a pair the dealer could buy 75 pairs. Since \$1 is $\frac{4}{5}$ of \$1.25, at \$1.25 he can buy $\frac{4}{5}$ of 75 pairs, or 60 pairs.

ORAL AND WRITTEN EXERCISES

4. How many articles costing $33\frac{1}{3}¢$ each can be bought for: *a.* \$7? *b.* \$20? *c.* $\$6\frac{1}{3}$? *d.* \$15? *e.* $\$49\frac{2}{3}$?

5. How many articles costing $16\frac{2}{3}¢$ can be bought for: *a.* \$12? *b.* \$30? *c.* $\$4\frac{1}{2}$? *d.* $\$8\frac{2}{3}$? *e.* $\$92\frac{1}{2}$?

6. How many articles costing $12\frac{1}{2}¢$ each can be bought for: *a.* \$5? *b.* $\$3\frac{1}{4}$? *c.* $\$5\frac{1}{2}$? *d.* $\$17\frac{1}{2}$? *e.* \$163?

7. How many articles at $\$1\frac{1}{2}$ each can be bought for: *a.* \$12? *b.* \$27? *c.* \$132? *d.* \$873? *e.* \$216?

8. How many articles at $66\frac{2}{3}¢$ each can be bought for: *a.* \$6? *b.* $\$2\frac{2}{3}$? *c.* \$60? *d.* \$174? *e.* \$368?

9. How many articles costing \$2.50 each can be bought for: *a.* \$10? *b.* \$25? *c.* \$175? *d.* \$87.50? *e.* \$115?

10. How many articles costing 50¢ each can be bought for: *a.* \$9? *b.* \$36? *c.* $\$18\frac{1}{2}$? *d.* \$216? *e.* $\$87\frac{1}{2}$?

11. How many articles costing 25¢ each can be bought for: *a.* \$6? *b.* \$2.75? *c.* \$12.50? *d.* \$114? *e.* \$846.25?

12. How many articles costing 20¢ each can be bought for: *a.* $\$9\frac{1}{5}$? *b.* \$17? *c.* \$142? *d.* \$494? *e.* \$278?

1. Find the cost of 786 ft. wire fence at $2 per C.

$2 } = 7.86
7.86 } = $2
$15.72

786 = 7.86 C. Since 1 C costs $2, 7.86 C will cost 7.86 times $2, or $15.72. To shorten the work we always use in practice the 2 as the multiplier. In explaining the work, however, be careful to use the $2 as the multiplicand.

2. Find the cost of 2860 ft. of lumber at $35 per M.

2.86
$35
$100.10

2860 = 2.86 M. Since 1 M costs $35, 2.86 M will cost 2.86 times $35. In practice use the smaller number as the multiplier.

3. Find the cost of 8760 lb. of coal at $6.25 a ton.

$6.25
4.38
$27.375

8760 lb. = 4.38 T. Since there are 2000 lb. in 1 T, in 8760 lb. there will be as many tons as 2000 lb. is contained times in 8760 lb. First divide by 1000 by pointing off 3 places. Then divide the quotient by 2. Since 1 T cost $6¼, 4.38 T will cost 4.38 times $6.25, or $27.38.

4. Find the cost of 12,400 lb. of hay at $18 a ton.

12.4
$9
$11.16

When the price of a ton is divisible by 2, this method is often used. 12,400 = 12.4 M. $18 for 2000 lb. is $9 per M. Since 1 M cost $9, 12.4 M will cost 12.4 times $9.

Find the cost of:

5. 3200 boxes of oranges at $225 per C.
6. 17,624 lb. of coal at $6.75 a ton.
7. 18,460 lb. of hay at $22 a ton.
8. 7878 ft. of lumber at $48 per M.
9. 210,200 cu. ft. of gas at 95¢ per C.
10. 24,150 ft. of birch flooring at $55 per thousand feet.
11. Find the freight on 48,500 lb. of cotton, shipped from Louisiana to New York at 61¢ per C.
12. If the freight is 35¢ per C, find the freight on 26,400 lb. of hides shipped from Chicago to Boston.
13. Find the freight on 1,176,000 lb. of coal, shipped from Pennsylvania to New Haven at $1.25 a ton.

1. A **denominate number** is a concrete number denoting measure or weight, as 3 pounds, 2 feet, 3 quarts.

2. A **unit of measure** is a definite unit, established by law or custom, used in measuring quantity. Thus 1 bushel, 1 foot, are units of measure, and 1 pound is a unit of weight.

3. A **simple denominate number** is a denominate number expressed in terms of one denomination, as 2 miles, 8 ounces, 3 gallons, etc.

4. A **compound denominate number**, usually called a **compound number**, is a denominate number expressing more than one denomination related to each other, as 1 yd. 2 ft.

5. **Reduction** is the changing from one form to another without changing its value, as 3 ft. 6 in. may be expressed as $3\frac{1}{2}$ ft., or as 42 in.

6. In changing 3 ft. 6 in. to 42 in., we change to a lower denomination. This process is called **reduction descending.**

7. In changing 42 in. to 3 ft. 6 in., we change to a higher denomination. This process is called **reduction ascending.**

NOTE: Tables of denominate numbers may be found on pages 256–258.

8. Change 5 bu. 2 pk. to quarts.

Analysis	Usual Method
1 bu. = 4 pk.	5 bu. 2 pk.
5 bu. = 5 × 4 pk. = 20 pk.	4
20 pk. + 2 pk. = 22 pk.	20 pk.
1 pk. = 8 qt.	2 pk.
22 pk. = 22 × 8 qt. = 176 qt.	22 pk.
	8
	176 qt.

Since 1 bu. = 4 pk., 5 bu. = 5 times 4 pk. = 20 pk. Since there are 20 pk. in 5 bu., in 5 bu. 2 pk. there will be 20 pk. + 2 pk. = 22 pk. Since there are 8 qt. in 1 pk., in 22 pk. there will be 22 times 8 qt. or 176 qt. In practice we multiply by 4 and 8 as shown in illustration, since the result will be the same.

1. Change 175 pt. to quarts and gallons.

2)175 pt.
4)87 + 1 pt.
21 + 3 qt.
21 gal. 3 qt. 1 pt.

Since 2 pt. = 1 qt., 175 pt. will equal as many quarts as 2 pt. is contained times in 175 pt. which is 87 times and 1 pt. remaining. Since 4 qt. = 1 gal., 87 qt. will equal as many gallons as 4 qt. is contained times in 87 qt., which is 21 times and 3 qt. remaining. The answer is 21 gal., 3 qt., 1 pt.

ORAL EXERCISES

Change:

2. 5 bu. to pecks.
3. 64 pk. to bushels.
4. 12 pk. to quarts.
5. 56 qt. to pecks.
6. 20 gal. to quarts.
7. 60 qt. to gallons.
8. 27 qt. to pints.
9. 120 pt. to quarts.
10. 2 mi. to rods.
11. 1280 rd. to miles.
12. 2 rd. to yards.
13. $16\frac{1}{2}$ yd. to rods.
14. 25 yd. to feet.
15. 96 ft. to yards.
16. 15 ft. to inches.
17. 144 in. to feet.
18. 36 ft. to fathoms.
19. 27 mi. to leagues.
20. 2 sq. mi. to acres.
21. 1920 A. to square miles.
22. 4 A. to square rods.
23. 800 sq. rd. to acres.
24. 2 sq. rd. to square yards.
25. 121 sq. yd. to square rods.
26. 40 sq. yd. to square feet.
27. 279 sq. ft. to square yards.
28. 4 sq. ft. to square inches.
29. 288 sq. in. to square feet.
30. 2 cd. to cubic feet.
31. 128 cu. ft. to cords.
32. 6 cu. yd. to cubic feet.
33. 81 cu. ft. to cubic yards.
34. 20 T. to pounds.
35. 8000 lb. to tons.
36. 6 lb. to ounces.
37. 80 oz. to pounds.
38. 12 yr. to months.
39. 240 mo. to years.
40. 8 da. to hours.
41. 96 hr. to days.
42. 8 score to units.
43. 80 quires to reams.
44. 48 sheets to quires.
45. 8 reams to quires.
46. 144 units to dozens.
47. 60 doz. to units.
48. 10 pounds to shillings.
49. 40 shillings to pence.

1. Change $\frac{1}{5}$ of a day to hours and minutes.

$\frac{1}{5}$ of 24 hr. $= 4\frac{4}{5}$ hr.
$\frac{4}{5}$ of 60 min. $= 48$ min.
Ans. 4 hr. 48 min.

Since there are 24 hr. in a day, in $\frac{1}{5}$ of a day there will be $\frac{1}{5}$ of 24 hr., or $4\frac{4}{5}$ hr. Since there are 60 min. in an hour, in $\frac{4}{5}$ of an hour there will be $\frac{4}{5}$ of 60 min., or 48 min.

2. Change .625 sq. mi. to square rods.
$.625 \times 640$ A. $= 400$ A.
400×160 sq. rd. $= 64{,}000$ sq. rd.

3. What part of a gallon is 3 qt. 1 pt.?

3 qt. 1 pt. $= 7$ pt.
1 gal. $= 8$ pt.
7 pt. $= \frac{7}{8}$ gal.

Change 3 qt. 1 pt. to pints. Change the gallon also to pints. The question now is, "7 pt. is what part of 8 pt.?"

4. Reduce 25,480 min. to a compound number.

5. A man bought 8 A. of land at \$175 an acre and sold it at 22¢ a square foot. What was the gain or loss?

6. What will 5 bu. 3 pk. 4 qt. of nuts cost at \$4.80 a bushel?

7. How much will it cost to fence $3\frac{3}{4}$ mi. of road at $62\frac{1}{2}$¢ a rod?

8. What part of 6 rd. is 3 yd.?

9. Change 3 bu. 1 pk. to quarts.

10. Change 976 qt. to bushels.

11. Reduce 3 da. 5 hr. 30 min. to minutes.

12. How many pounds are there in $37\frac{1}{2}$ long tons?

13. Reduce $25\frac{3}{4}$ lb. to ounces.

14. Change 32,176 oz. to higher denominations.

15. Change $4\frac{1}{2}$ sq. ft. to the next lower denomination.

16. Reduce 816 pt. to the next higher denomination.

17. Reduce 280 cd. to cubic feet.

18. Change 317 cu. yd. to the next lower denomination.

19. Reduce 5 A. to square feet.

20. What part of 1 sq. ft. is $2\frac{1}{2}$ sq. in.?

21. What part of a circumference is an angle of 45°?

1. Add 15 da. 20 hr. 40 min. and 7 da. 10 hr. 30 min.

15 da. 20 hr. 40 min.
7 da. 10 hr. 30 min.
22 da. 30 hr. 70 min.
23 da. 7 hr. 10 min.

Add each column separately. The result is 22 da. 30 hr. 70 min. Since 70 min. = 1 hr. 10 min., we write 10 min. in the column of minutes and unite the 1 hr. with 30 hr., making 31 hr. Since 31 hr. = 1 da. 7 hr., we write 7 hr. in the column of hours and unite the 1 da. with 22 da., making 23 da.

2. From 15 yd. 6 in. take 9 yd. 8 in.

15 yd. 6 in.
9 yd. 8 in.
5 yd. 34 in.

Since we cannot take 8 in. from 6 in., we take 1 yd. and change it to inches, and unite it with 6 in., making 42 in. 42 in. − 8 in. = 34 in. 14 yd. − 9 yd. = 5 yd.

Add:

3. 6 da. 17 hr.
3 da. 11 hr.
5 da. 16 hr.

4. 4 hr. 36 min. 18 sec.
5 hr. 18 min. 41 sec.
8 hr. 45 min. 20 sec.

5. 17 lb. 10 oz.
8 lb. 6 oz.
47 lb. 11 oz.

6. 7 yd. 2 ft.
9 yd. 1 ft.
6 yd. 0 ft.

7. 6 mi. 100 rd. 2 yd.
5 mi. 120 rd. 4 yd.
8 mi. 90 rd. 3 yd.

8. 7 A. 42 sq. rd.
8 A. 30 sq. rd.
5 A. 90 sq. rd.

9. 7° 37′
9° 25′
4° 50′

10. 8° 37′ 48″
2° 26′ 30″
5° 39′ 40″

11. 7 cu. ft. 875 cu. in.
6 cu. ft. 574 cu. in.
8 cu. ft. 908 cu. in.

Subtract:

12. 25 A. 60 sq. rd.
15 A. 120 sq. rd.

13. 27 yd. 1 ft. 4 in.
15 yd. 2 ft. 9 in.

14. 18 sq. yd. 3 sq. ft.
9 sq. yd. 7 sq. ft.

15. A note was given July 23, 1905, and paid Sept. 10, 1911. How long did it run?

1911 − 9 − 10
1905 − 7 − 23
6 1 17
yr. mo. da.

Write the dates as shown in the illustration. Since we cannot take 23 days from 10 days, we take a month, and calling it 30 days, unite it with the 10 days.

16. From Jan. 1, 1910, take Dec. 11, 1904.

1. *a.* Multiply 28 ft. 5 in. by 5.

$$\begin{array}{r} 28 \text{ ft. } 5 \text{ in.} \\ \underline{\qquad\quad 5} \\ 140 \text{ ft. } 25 \text{ in.} \\ \underline{142 \text{ ft. } 1 \text{ in.}} \end{array}$$

Multiply each denomination separately. Beginning with the lowest denomination, change, when possible, to the next higher denomination, as in addition of compound numbers.

b. Multiply 6 ft. 4 in. by 4 ft. 3 in.

6 ft. 4 in. = 76 in. or $6\frac{1}{3}$ ft.
4 ft. 3 in. = 51 in. or $4\frac{1}{4}$ ft.
A. (76 × 51) sq. in. = 3876 sq. in.
3876 sq. in. = 26 sq. ft. 132 sq. in.
B. $(6\frac{1}{3} \times 4\frac{1}{4})$ sq. ft. = $26\frac{1}{12}$ sq. ft.

To multiply one compound number by another, reduce both to the same denomination before multiplying. In A both were changed to square inches. The product is 3876 sq. in., which equals 26 sq. ft. 132 sq. in. In B both were changed to square feet. The result is $26\frac{1}{12}$ sq. ft.

Multiply:

2. 15° 36′ 46″ by 15.
3. 4 yd. 2 ft. 8 in. by 6.
4. 2 bu. 3 pk. 5 qt. by 8.
5. 9 hr. 35 min. 30 sec. by 9.
6. 8 mi. 960 ft. by 36.
7. 18 T. 1700 lb. by 18.
8. 3 rd. 5 yd. by 5 yd. 1 ft.
9. 5 ft. 6 in. by 4 ft. 9 in.

10. A tract of land was divided into 35 lots, each containing 3 A. 40 sq. rd. What was the area of the tract?

11. A farmer sold 15 loads of hay, averaging 1 T. 1200 lb. in weight. Find the total weight of the hay sold.

12. The fence on one side of a square field measures 20 rd. 5 yd. Find the distance round the field.

13. If a school has in the morning 4 recitations, each of 45 min., and in the afternoon 3 recitations, each of 35 min., how many hours and minutes of recitations are there in a day?

14. If a woman can stitch each garment in 1 hr. 45 min., how long will it take her to stitch 15 garments?

15. The average yield of coffee to each tree is 1 lb. 10 oz. Find the total yield from a plantation of 15.5 A., if there are 480 trees on each acre.

1. Divide 11 ft. 8 in. by 4.

A.
$$4\overline{)11 \text{ ft.} - 8 \text{ in.}}\quad 2 \text{ ft.} - 11 \text{ in.}$$

B.
$$4\overline{)11 \text{ ft.} - 8 \text{ in.}}\quad 2 \text{ ft.} - 11 \text{ in.}$$
$$8 \text{ ft.}$$
$$3 \text{ ft.}$$
$$12$$
$$36 \text{ in.}$$
$$8 \text{ in.}$$
$$44 \text{ in.}$$
$$44 \text{ in.}$$

A shows the method of dividing a compound number by an integer using short division. In B all the work is shown as in long division.

$\frac{1}{4}$ of 11 ft. is 2 ft., and 3 feet remaining. Change the 3 ft. remaining to 36 in., and unite it with 8 in., making 44 in. $\frac{1}{4}$ of 44 in. is 11 in.

2. Divide 22 lb. 8 oz. by 2 lb. 4 oz.

22 lb. 8 oz. = $22\frac{1}{2}$ lb., or 360 oz.
2 lb. 4 oz. = $2\frac{1}{4}$ lb., or 36 oz.
A. $22\frac{1}{2}$ lb. $\div$ $2\frac{1}{4}$ lb. = 10.
B. 360 oz. $\div$ 36 oz. = 10.

In dividing one compound number by another, change both numbers to the same denomination before dividing. Notice that it makes no difference whether we change to pounds or ounces.

Divide:

3. 90 bu. 3 pk. by 4.

4. 78 T. 1500 lb. by 15.

5. 116 yd. 2 ft. by 9.

6. 75 cu. yd. 12 cu. ft. by 24.

7. 41 gal. 2 qt. by 2 gal. 1 qt.

8. 35 yd. 1 ft. by 6 yd. 2 ft.

9. If a man walks at the rate of a mile in 18 min., how many miles can he walk in 1 hr. 36 min.?

10. If a man can build 15 ft. 4 in. of stone wall in a day, how many days will it take him to build 107 ft. 4 in.?

11. A man's farm contained 28 A. 20 sq. rd. He divided it into building lots, each containing 1 A. 14 sq. rd. How many building lots were there?

12. How many steps will you take in walking $\frac{1}{2}$ mile, if each step is 2 ft. 6 in. in length?

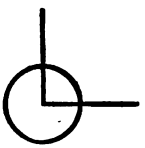
13. How many times can a 2 quart can be filled from a tank of gasolene containing 44 gal. 2 qt.?

1. **Mensuration** is the process of computing the lengths of lines, the areas of surfaces, and the volumes of solids.

2. A **straight line** is a line that does not change its direction. A **curved line** is a line that changes its direction at every point. **Parallel lines** are lines which have the same direction.

3. An **angle** is the difference in direction of two lines.

4. Angles are measured by degrees. How many degrees are there in a circle?



5. This angle is measured by what part of the circle? It is an angle of 90°, or a **right angle.**

6. An **acute angle** is an angle smaller than a right angle.

7. An **obtuse angle** is an angle greater than a right angle.

8. Using a protractor, make an angle of 30°. Of 60°. Of 150°. Of 110°.

9. A **polygon**, meaning many angles, is a plane figure bounded by straight lines.

10. **Perimeter** is the distance round any plane figure.

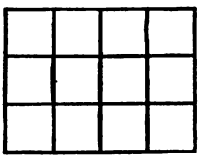
11. A **triangle** is a polygon of 3 sides, and a **quadrilateral** is a polygon of 4 sides.

12. Though you will seldom use them, it may be well to know that a figure of 5 sides is a **pentagon**; of 6 sides, a **hexagon**; of 8 sides, an **octagon**; of 9 sides, a **nonagon**; of 10 sides, a **decagon.**

13. Quadrilaterals are named from the relations of their sides or from their angles, as a square, a rectangle, a parallelogram, a rhomboid, a rhombus, a trapezoid, a trapezium.

14. A **square** is a figure having 4 equal angles and 4 equal sides. A **rectangle** is a figure having 4 equal angles and opposite sides equal.

1. When the opposite sides of a quadrilateral are equal and parallel, it is also called a **parallelogram.**



2. Find the area of a rectangle 4 in. long and 3 in. wide. Since there are 4 sq. in. in one row, in 3 rows there will be 3 times 4 sq. in. or 12 sq. in.

Learn: To find the area of a parallelogram multiply the number of square units in one row by the number of rows.

NOTE: For the sake of brevity we usually say, to find the area of a parallelogram multiply the length by the width.

3. Since the area of a parallelogram is equal to the product of the two dimensions, it follows that if the area and one dimension are given, the other may be found.

4. Let S = the surface or area, b the base, and a the altitude. We have three general formulas:

$$S = ab. \quad a = S \div b. \quad b = S \div a.$$

5. State these formulas as rules.

6. What is the value at \$60 an acre of a tract of land 80 rd. long and 60 rd. wide?

7. A man cut a street $\frac{1}{2}$ mile long and 60 ft. wide through his land. How many acres did he use?

8. A man bought a field of $10\frac{1}{2}$ acres. From this he made 15 house lots of 10,800 sq. ft. each, and the rest he divided equally into 24 lots. How many square feet were there in each of the 24 lots?

9. A floor measures 18 ft. by 21 ft. How many square yards does it contain?

10. If there are 40 sq. yd. in the area of a room that is 15 ft. wide, how long is the room?

11. How many blocks will it take to pave a street $\frac{1}{2}$ mi. long and 33 ft. wide, if it takes 45 blocks to pave a square yard?

ROOFING

Shingles are put up in bunches, and 4 bunches make 1000. Shingles are usually laid 4 in. to the weather, and it is estimated that 1000 will cover a square (100 sq. ft.). A fraction of a bunch cannot be bought.

1. At \$4.25 per M how much will the shingles cost for the roof of a house 34 ft. long, if the rafters are 22 ft. long?

$2\ (34 \times 22)$ sq. ft. $= 1496$ sq. ft.
1496 sq. ft. $= 14.96$ squares $\therefore$ 15 M.
\$4.25 $\times$ 15 = \$63.75.

2. The roof of a house is 40 ft. long, and the rafters are 24 ft. long. At \$4.75 per M how much will the shingles cost for both sides of the roof?

3. What will be the cost of shingles at \$2.75 per M to cover the two sides of a roof, each side measuring 32 ft. by 18 ft.?

LATHING AND PAVING

Laths are usually put up in bundles of 100 each, and are estimated to cover 6 square yards. In estimating quantity, deduct $\frac{1}{2}$ of the area of all openings.

4. How many bundles of laths will be required for the ceiling of a room 19 ft. by 16 ft.?

19 sq. ft. $\times$ 16 $= 304$ sq. ft. $= 33\frac{7}{9}$ sq. yd.
$33\frac{7}{9}$ sq. yd. $\div$ 6 sq. yd. $= 5\frac{17}{7}$ $\therefore$ 6 bundles.

5. If the price of laths is 40¢ a bundle and it costs 15¢ a bundle to put them on, find the cost of lathing a room 14 ft. long, 12 ft. wide, and 9 ft. high. There are 2 doors $6\frac{1}{2}$ ft. by 4 ft. and 3 windows 6 ft. by $3\frac{1}{2}$ ft.

6. At 27¢ a square foot, find the cost of paving a walk 5′ by 105′.

7. At \$1.75 a square yard, what will it cost to put a 5-foot sidewalk round the outside of a park 200 ft. long and 180 ft. wide?

1. When you know the length of one side of a square, how do you find its perimeter and area?

2. When you know the perimeter of a square, how do you find the length of each side?

3. When you know the base and area of a triangle, how do you find the altitude?

4. What do you mean by quadrilateral?

5. Give the names of 5 different quadrilaterals.

6. What two dimensions of a rectangle do you need to know to find its area?

7. When you know the area and length of a rectangle, what can you find? How do you find it?

8. What would you need to know to find the number of yards of carpet for this floor?

9. What dimensions would you need to know to find the number of square yards of plastering in this room?

10. State the rule for finding the number of rolls of paper needed for the walls and ceiling of any room.

11. How would you find the number of bunches of laths needed for this room?

12. How do you find the number of board feet in any given board or plank?

13. Find the area of a triangle whose base is 10 ft. and altitude 6 ft.

14. The area of a triangle is 24 sq. ft. If the altitude is 6 ft., what is the base?

15. How many square feet are there in a rhomboid whose base measures 8 ft., and whose height is 4 ft.?

16. The area of a parallelogram is 120 sq. ft. If the base is 12 ft., what is the altitude?

17. The area of a field is 10 acres. If its length is 80 rd., what is its width?

18. A floor 27 ft. by 18 ft. contains how many square yards?

1. A **solid** is an object that has length, breadth, and thickness. The contents of a solid is called its **volume.**

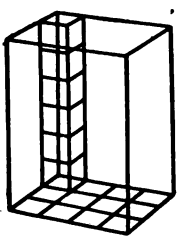
2. A **rectangular solid**, also called a **rectangular prism**, is a solid that is bounded on all sides by rectangles.

3. The rectangles that bound the solid are called the **faces.**

4. A **cube** is a rectangular solid whose faces are squares.

5. A **triangular prism** is a solid bounded by three rectangular faces having for its bases two triangles that are parallel to each other.

6. A **cylinder** is a solid bounded by a uniformly curved surface and having for its bases two circles that are parallel to each other.



7. How many cubic inches are there in a prism 4 in. long, 3 in. wide, and 6 in. high?

a. Multiply the number of cubic inches in one row by the number of rows, to find the number of cubic inches in one layer.

b. Multiply the number of cubic inches in one layer by the number of layers, to find the number of cubic inches in the prism.

8. Find the number of cubic inches in a cylinder that is 4 in. high, if the base contains 8 sq. in.



If the base contains 8 sq. in., then we can place 8 cu. in. in one layer 1 in. high on the base. If the cylinder is 4 in. high, there would be 4 layers. Hence the cylinder must contain 4 times 8 cu. in., or 32 cu. in.

Learn: To find the volume of a cylinder or any regular prism, multiply the area of the base by the altitude.

NOTE: This rule is usually given for the sake of brevity. In reality we multiply the number of cubic units that can be placed on the base by the number of units in altitude.

1. Let l = length, V = volume, then the formula for finding the volume of a cylinder is $V = l\pi R^2$.

2. Find the volume of a cylinder when the radius of the base is 8 in. and the length is 21 in.

3. Find the volume of a cylindrical iron vat $4\frac{1}{2}$ ft. in diameter and 10 ft. deep.

4. Find the volume of a rectangular prism whose altitude is 40 ft. when the sides of the base are 9 ft. and 5 ft.

5. Find the capacity in cubic feet of a cylindrical cistern 6 ft. in diameter and 9 ft. deep.

6. How many cubic feet are there in a cylinder 60 ft. long and 8 ft. in diameter?

7. How many gallons of water are in a cistern 6 ft. in diameter if the water is 4 ft. deep?

NOTE: A gallon contains 231 cu. in. or approximately $7\frac{1}{2}$ gal. equal 1 cu. foot, and $4\frac{1}{2}$ cu. ft. equal 1 barrel. Hence to find the number of gallons in a cistern:

a. For practical purposes multiply the number of cubic feet in the cistern by $7\frac{1}{2}$.

b. For exact purposes divide the number of cubic inches in the cistern by 231.

8. The surface of a lake is $4\frac{1}{2}$ sq. mi. How many gallons of water will it take to raise the surface $\frac{1}{2}$ in.?

9. How many gallons are in a tub having a base of $3\frac{1}{2}$ sq. ft. and a depth of 15 in.?

10. Find the volume of a square prism, if the perimeter of the base is 44 ft. and the altitude 42 ft.

11. The ice on a circular pond is 18 in. thick. If the pond is 800 ft. in circumference, how many cubic feet of ice does it contain?

12. Wishing to find the number of cubic inches in an irregular piece of stone, I sunk it in a cylindrical glass of water, and found it raised the water $3\frac{1}{4}$ in. If the glass was 6 in. in diameter, how many cubic inches were there in the stone?

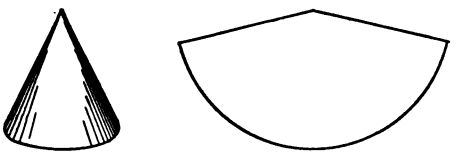
1. The **altitude** of a cone or pyramid is the perpendicular distance from the vertex to the base.

2. The **slant height** of a pyramid is a straight line from the vertex to the center of one side of the base.

3. The **slant height** of a cone is a straight line from the vertex to the circumference of the base.

NOTE: In this book all cones are supposed to be right circular cones.

4. Make a paper cone. Cut it on a line running from any point in the circumference of the base to the vertex. Open it as shown in Fig. 2. You have a form so much like a triangle that its area may be found in the same way as the area of a triangle.



5. Find the convex surface of a cone, when the circumference of the base is 10 ft. and slant height 20 ft.

$\frac{1}{2}$ (20 × 10) sq. ft. = 100 sq. ft.

6. Find the lateral surface of a square pyramid, whose base measures 6 in. and slant height 10 in.

4 × 6 in. = 24 in. perimeter.

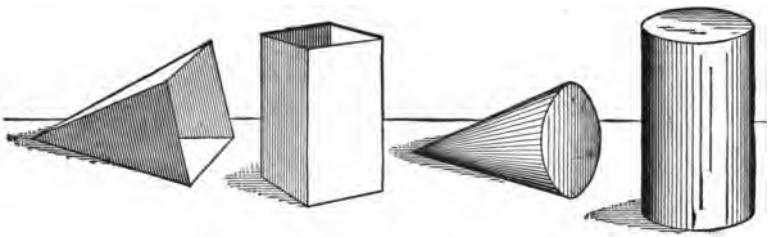
$\frac{1}{2}$ (24 × 10) sq. in. = 120 sq. in.

Learn: To find the convex surface of a cone or pyramid, multiply the distance round the base by $\frac{1}{2}$ of the slant height. To this sum add the area of the base to find the entire surface.

7. How many square yards of canvas will it take to make a conical tent 24 ft. in diameter and whose slant height is 18 ft.?

8. At 35¢ a square yard find the cost of painting a church spire in the form of a pyramid; whose base is a 6 ft. octagon, and slant height 75 ft.

9. Find the entire surface of a cone whose circumference is 22 ft., and slant height 68 ft.



NOTE: There should be in the room a hollow cone and cylinder, each having the same base and altitude, also a hollow pyramid and prism, each having the same base and altitude. These can easily be made from cardboard. Let the pupils discern that the volume of the cone and pyramid is in each case $\frac{1}{3}$ of the volume of the cylinder and prism having the same base and altitude.

1. Find the volume of a square pyramid, each side of whose base is 4 ft., and whose altitude is 18 ft.

2. Find the volume of a cone, the circumference of whose base is 9 ft., and whose altitude is 14 ft.

3. Find the volume of a square pyramid, the perimeter of whose base is 12 ft., and whose altitude is 12 ft.

4. Find the volume of a cone whose diameter is 6 ft., and altitude 18 ft.

5. What is the volume of a cone whose altitude is 42 in., and whose diameter is 24 in.?

6. Find the number of bushels of oats in a conical pile whose base covers 57 sq. ft., and whose altitude is 15 ft.

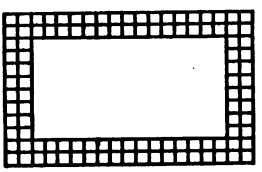
7. Find the volume of a pyramid, the area of whose base is 30 sq. ft., and whose altitude is 18 ft.

8. The altitude of a cone is 20 in. and the radius of its base 15 in. Find its volume.

9. How many cubic inches are there in a square pyramid the perimeter of whose base is 4 ft. 8 in., and whose altitude is 2 ft. 8 in.?

10. In using our lantern, the rays of light from the lens form a cone of light. If the curtain is 30 ft. from the lens and the diameter of the circle on the curtain is 12 ft., how many cubic feet of air are in the cone of light?

In measuring for cellars and buildings, masons measure the distance round the outside of the walls, thus counting each corner twice. This is considered fair because of the greater labor in constructing the corners. For the same reason no allowance is made for an opening unless it is a large one, and then $\frac{1}{2}$ is deducted. 22 bricks with mortar are allowed for each foot of wall.



1. How many bricks will be needed for the walls, 2 ft. thick, of a house 80 ft. long, 50 ft. wide, and 18 ft. high?

2 (80 + 50) ft. = 260 ft. perimeter.
(260 × 18 × 2) cu. ft. = 9360 cu. ft.
22 bricks × 9360 = 205,920 bricks.

2. How many bricks are required for the 8 in. walls of a rectangular cistern, the outside dimensions of which are, length 8 ft., width 8 ft., depth 6 ft.?

3. A brick house is 32 ft. long, 28 ft. wide, and 24 ft. high. There are 24 windows 3 ft. by 6 ft., and 3 doors 4 ft. by 7 ft. If the walls are 18 in. thick, how much will the bricks cost at $4.25 per M?

4. A house measures 48 ft. by 32 ft. If the cellar wall is 7 ft. high and 18 in. thick, find the cost of building it at 40¢ a cubic yard.

5. Above the cellar wall in example 4 is a brick wall 2 ft. high and 16 in. thick. Find the cost of the bricks at $3.50 per M.

6. How many bricks will be required to build a house 36 ft. long, 28 ft. wide, and 20 ft. high, if the wall is $1\frac{1}{3}$ ft. in thickness and has 4 doors, 4 ft. by 8 ft., and 32 windows, 3 ft. by 6 ft.?

7. At $5.25 a thousand find the cost of bricks for a building 45 ft. long, 33 ft. wide, 22 ft. high, if the walls are $1\frac{1}{2}$ ft. thick. There are 6 windows 7 ft. by 3 ft., and 4 doors 4 ft. by $8\frac{1}{2}$ ft.

1. A **sphere** is a solid bounded by a convex surface, every point of which is equally distant from a point within, called the center.

a. Place a sphere on the table. Holding it between the palms of the hands roll it over once. Do you see that you have covered a rectangle as long as the circumference of the sphere and as wide as the diameter?





b. Wind half of a sphere with a cord, as shown in the illustration, then with the same cord wind the convex surface of a cylinder whose diameter and altitude are each the same length as the diameter of the sphere. The cord will cover one half of the convex surface of the cylinder, hence the surface of a sphere equals the convex surface of a cylinder, having the same diameter and height.

Learn: To find the surface of a sphere, multiply its diameter by its circumference.

2. Find the surface of a sphere whose radius is 6 ft.
3. Find the surface of a sphere whose diameter is 10 ft.
4. Find the volume of a sphere whose diameter is 4 ft.

a. By looking at a cut up wooden sphere, you will see that a sphere may be regarded as composed of pyramids, whose bases, taken together, form the surface of a sphere, and whose altitude is the radius of the sphere.

b. By means of a sphere and a hollow cylinder it may be shown that the volume of the sphere equals two thirds the volume of a cylinder whose diameter and altitude are each the same length as the diameter of the sphere.

Learn: To find the volume of a sphere:

a. Multiply the surface by one third of the radius.

b. Find two thirds the volume of a cylinder of the same diameter and height.

1. State the rule for finding the entire surface of a cylinder.

2. Find the volume of a cylinder, when the altitude is 10 in. and the area of the base is 15 sq. in.

3. How does the volume of a cone compare with the volume of a cylinder, having the same base and altitude?

4. What dimensions of a cone do you need to know to find its convex surface?

5. How many 2 in. cubes can be placed in a cubical box whose length is 10 in.?

6. A pile of wood is 40 ft. long, 4 ft. wide, and 4 ft. high. How many cords does it contain?

7. How do you find the approximate number of gallons of water in a cistern?

8. How would you find the exact number of gallons in a cistern?

9. How many bricks are allowed for each cubic foot of wall?

10. How do masons usually measure the walls of a building in estimating the amount of labor in constructing it?

11. How do you find the entire surface of a sphere?

12. Find the number of square inches in the surface of a sphere 7 in. in diameter, using $\pi = 3\frac{1}{7}$.

13. Define a circle.

14. Define a parallelogram.

15. How can you find the diameter of a tree?

16. How would you find the number of bushels of corn that can be placed in any bin?

17. If you know the number of bushels of wheat in a pile, how can you estimate the size of the bin needed to hold it?

18. A bundle of laths is estimated to cover how much surface?

19. A thousand shingles is estimated to cover how much surface?

Find the surface of spheres, when the following dimensions are known:

1. Radius 5 in.
2. Diameter 8 in.
3. Circumference 31.416. in.
4. Diameter 2 ft. 6 in.
5. How much leather will cover a base ball if its diameter is $3\frac{1}{2}$ in.?
6. At 15¢ a square foot, what will it cost to gild a sphere 14 in. in diameter? Use $\pi = 3\frac{1}{7}$.
7. The radius of a sphere is 15 in. Find its volume.
8. A man cut a cylinder of the largest possible size out of a cubical block of wood measuring 2 ft. What part of the cube did he cut away?
9. If the man should cut a sphere from the same sized cube, what part would he cut away?
10. From a sphere, 15 in. in diameter, two inches in thickness were cut off. How many cubic inches were cut off?
11. If the diameter of the earth were exactly 8000 miles, what would be its volume? How many square miles would there be in its surface?
12. A cubic foot of iron weighs 450 lb. Find the weight of a solid cannon ball 14 in. in diameter.
13. My house is on a corner lot, 150 ft. on one street and 60 ft. on the other. If the sidewalk is 6 ft. wide, how many cubic feet of snow do I shovel in clearing my walk after a 15 in. snowstorm?
14. A cistern is 4 ft. in diameter and 6 ft. deep. What will it cost to line it with lead, weighing 5 lb. to the square foot, and costing $9\frac{1}{2}$¢ a pound?
15. How many boards 15 ft. long will be required to build a fence 5 boards high about a rectangular field 40 rd. by 10 rd.?
16. The base of a triangle is 60 ft. and the altitude 40 ft. Find the base of a triangle with the same altitude, and area twice as large.

1. The perimeter of a square measures 28 inches. What is its area?

2. The perimeter of a rectangle is 36 in. If the length is twice the width, find the area of the rectangle.

3. The perimeter of a rectangle is 24 in., and the width is 5 in. Find the length and area.

4. What dimensions do you need to know to find the area of a trapezoid? State the rule.

5. The area of a trapezoid is 72 sq. in., and the parallel sides are 10 in. and 8 in. What is the width of the trapezoid?

6. Find the area of a triangle, having a base of 30 ft. and an altitude of 12 ft.

7. State the rule for finding the circumference of a circle when you know its radius.

8. State the rule for finding the area of a circle when you know its circumference.

9. At \$60 an acre find the cost of a rectangular field 40 rd. long and 8 rd. wide.

10. How many tiles 6 in. square will it take to pave a vestibule that measures 10 ft. by 5 ft.?

11. Find the number of board feet in a plank 20 ft. long, 1 ft. wide, and 3 in. thick.

12. Find the board feet in a board 18 ft. long, 12 in. wide at one end, and 8 in. wide at the other end. First find the average width of the board.

13. A room measures 15 ft. by 12 ft. If the breadths run lengthwise and measure 1 yd. in width, how many yards of carpet will be needed for the room?

14. A room is 12 ft. long, 10 ft. wide, and 8 ft. high. How many square feet of plastering are there in the ceiling? In one side? In one end?

15. Find the entire surface of a square prism 10 in. high, if the base is 2 in square. Find also its volume.

1. A pile of wood, 28 ft. long and 4 ft. wide, must be how high to contain 7 cords?

2. A man owns a rectangular field 60 rd. long and 8 rd. wide. If this field is divided into 8 equal rectangular lots, having their fronts on the longest side of the field, what will be the cost at \$2.25 a rod of fencing to inclose and separate the lots?

3. What is the weight of a granite roller $4\frac{1}{2}$ ft. in diameter and 8 ft. long? Granite is 2.62 times as heavy as water, and 1 cubic foot of water weighs $62\frac{1}{2}$ pounds.

4. What part of the volume of a square prism must be cut away to make a square pyramid of the same base and height as the prism? Prove it by a prism 4 ft. square and 8 ft. long.

5. At \$180 an acre find the cost of a field in the shape of a trapezoid whose parallel sides are 24 rd. and 18 rd., and width is 8 rd.

6. Find the cubic contents of a sphere whose circumference is 18 ft.

7. Which is greater and how much, the circumference of a circle whose diameter is 18 ft., or the perimeter of a square whose side is 14 ft.?

8. One side of a quadrilateral field is 60 rd., the side opposite and parallel to it is 40 rd., and the width is 32 rd. At \$125 an acre, what is the field worth?

9. The hot air register in our school-room is 3 ft. by 2 ft., and $\frac{1}{3}$ of the area is taken up by the grating. How much air a minute must pass through each square foot of this register to supply each of 48 pupils with 4 cu. ft. of fresh air every minute?

10. The distance round a circular park is $1\frac{1}{2}$ miles. How many acres does the park contain?

11. Compare the surface and volume of a cube whose edge is 3 ft. with the surface and volume of a cube whose edge is 6 ft.

1. Draw a horizontal line, AB, 2 in. long. At the point A, make an angle of 60° by drawing AC 3 in. long. Join B and C. What figure have you?

2. Draw another horizontal line, AB, 2 in. long. Make an angle of 110° at A, by drawing the line AC. Complete the triangle. Why do we call this figure an obtuse triangle?

3. Construct a right triangle.

4. Draw two lines, AB and AC, each 3 in. long, forming an angle of 45° at A. Join B and C. This is an isosceles triangle.

5. An isosceles triangle is a triangle having two of its sides equal.

6. Draw an isosceles triangle that shall contain an obtuse angle.

7. Draw a triangle. With your protractor measure each angle. Add the sum. What is the result?

8. Learn: The sum of the three angles of a triangle is equal to two right angles, or 180°.

9. Can you draw a triangle that shall contain an acute, an obtuse, and a right angle?

10. Draw the line AB. At A make an angle of 50°, and at B an angle of 60°. Prolong the lines until they meet at C. How many degrees are there in the angle at C?

11. Draw a triangle having at the base two angles of 60° each. How many degrees are there in the third angle? How do the sides of the triangle compare?

12. Since the three sides of this triangle are equal, we say it is an equilateral triangle.

13. Learn: Every equilateral triangle is equiangular.

14. Why must each angle of an equilateral triangle measure 60°?

15. What are the other angles when one angle of a right triangle is 35°? 40°? 30° 30′? 20° 20′ 20″? 27° 10′? 18° 20′? 28° 40′?

1. In an isosceles triangle the angle at the vertex is 40°. Find the angles at the base.

2. One acute angle of a right triangle is 40°. What is the other acute angle?

3. One angle at the base of an isosceles triangle is 60° 30′. Find the angle at the vertex.

4. What statement can you make about a triangle, when you know that its three sides are equal?

5. When the three angles of a triangle are equal, what is true of its sides?

6. When two acute angles of a right triangle are equal, how many degrees are there in each angle?

7. One angle of a triangle is 100°. How many degrees are there in each of the other angles if they are equal?

8. Cut a paper isosceles triangle. Fold it so as to show that the perpendicular from the vertex divides an isosceles triangle into two equal right triangles.

9. Draw: *a*. An acute isosceles triangle. *b*. An obtuse isosceles triangle. *c*. A right isosceles triangle.

10. Can you draw a triangle that contains a right and an obtuse angle? If not, why not?

11. Can you draw a triangle that contains two right angles? If not, why not?

12. Can you draw a triangle that contains a right and an acute angle? If not, why not?

13. Draw a right triangle whose sides are 3 in., 4 in., and 5 in.

14. Why is this problem impossible: Draw a triangle having one side 10 inches, and angles of 80° and 120°?

15. Can you draw a triangle with the sides respectively 9 in., 6 in., and 2 in.? If not, why not?

16. Find the angle at the vertex, if one of the base angles of an isosceles triangle is:

a. 42° 10′ 20″ *b*. 80° 15′ 24″

1. The perimeter of an equilateral triangle is 8 yd.; what is the length of each side in feet?

2. The perimeter of an isosceles triangle is 19 in. The unequal side is 7 in. Find the lengths of the other sides.

3. What is the radius of a circle whose circumference is 44 ft.?

4. $\frac{4}{11}$ of 77 are $\frac{4}{5}$ of what number?

5. If the sum of $\frac{1}{2}$ and $\frac{2}{3}$ is added to $\frac{1}{12}$, the sum will be how much more than one?

6. A man paid \$9 for a load of coal at \$6 a ton. How many pounds of coal were there in the load?

7. How many board feet are there in 10 boards, each 16 ft. long and 6 in. wide?

8. How many times is $\frac{3}{8}$ in. contained in $2\frac{1}{2}$ inches?

9. When the divisor is $\frac{2}{5}$ and the quotient is $\frac{5}{8}$, what is the dividend?

10. A man lost \$230. If he had $\frac{1}{3}$ of his money left, how much had he at first?

11. If 24 is $\frac{3}{8}$ of a number, what is $\frac{7}{8}$ of the number?

12. Mr. Brown purchased a horse, harness, and sleigh. The horse cost $\frac{3}{4}$ and the harness $\frac{1}{12}$ of the entire cost. If the price of the sleigh was \$30, what was the price of the horse and of the harness?

13. By selling 6 yd. of cloth for \$20 a merchant gained $\frac{1}{9}$ of what the cloth cost. What did it cost a yard?

14. What will 5 gal. of molasses cost, if 6 pt. cost 45 cents?

15. If 9 lb. of oatmeal cost \$.45, what will 27 lb. cost?

16. If 15 cords of wood are worth \$90, how much are $3\frac{2}{3}$ cd. worth?

17. What part of 3 is $\frac{1}{2}$ of $\frac{1}{2}$?

18. A room is 20 ft. long, 10 ft. wide, and 10 ft. high. How many square feet are there in the walls and ceiling?

19. Jennie has a piece of ribbon 15 in. long that contains 45 square inches. How wide is the ribbon?

1. At $\$6\frac{3}{4}$ a cord, find the cost of a pile of wood 20 ft. long, 4 ft. wide, and 6 ft. high.

2. A bushel of wheat weighs 60 lb. Find the value of 16,840 lb. of wheat at $98¢$ a bushel.

3. At \$12.75 a square foot what will it cost to gild the hemispherical dome, 40 ft. in diameter, on the State capitol?

4. At each corner of a square, 50 ft. on a side, with a radius of 25 ft. arcs of circles are drawn. Find the area of the part of the square not included within the circular part. With a scale of 1 inch to 10 ft. draw a diagram.

5. A cylinder is 8 in. in diameter and 28 in. in length. Find the volume of the largest cone that can be cut from it. How many cubic inches of the cylinder must be cut away?

6. How many bunches of shingles, laid 4 in. to the weather, will cover a roof, each half of which is 40 ft. by 20 ft.?

7. The last reading of my gas meter was 54,500 cu. ft. The previous reading was 47,900. At \$1.10 a thousand, find the amount of my gas bill.

8. At $\$1.12\frac{1}{2}$ a yard, what will it cost to carpet a flight of stairs of 18 steps, 12 in. tread, and 6 in. rise, allowing 2 in. on each step for turning, and $\frac{3}{4}$ yd. extra for both the top and the bottom?

9. The driving wheels of a locomotive are 15 ft. 9 in. in circumference. How many revolutions will they make in a mile? If the wheels revolve $2\frac{1}{2}$ times a second, what is the rate of speed a mile?

10. From a pile of wood 16 ft. long, 4 ft. wide, and 8 ft. high, there were sold at one time $2\frac{1}{4}$ cd., and at another time $1\frac{1}{2}$ cd. What is the remainder worth at \$5.75 a cord?

11. A bushel of corn weighs 56 lb. and a bushel of wheat 60 lb. How many bushels of wheat will weigh as much as 445 bu. of corn?

12. Find the cost of a load of lumber consisting of 40 planks, 16 ft. long, 8 in. wide, and 3 in. thick, at \$16 per M.

You learned in Lesson 86 that a state, county, and town needed money to pay their expenses. In this lesson we are to inquire about the expenses of our national government, and learn from what source it receives its income.

1. The government receives its money for all its expenses from:

a. An **Internal Revenue**, a tax on the right to make liquors, tobacco, etc.

b. **Customs**, or **Duties**, taxes on imported goods.

2. From your geography make a list of the more important things that are imported, and the name of the country from which they are imported.

3. Imported goods may be:

a. On the **free list**, admitted without duty.

b. Subject to **ad valorem duty**, a duty reckoned at a certain per cent on the value of the goods.

c. Subject to a **specific duty**, which is based on the number, quantity, and weight of the merchandise.

d. Subject to both specific and ad valorem duties.

4. Allowances, called tare and leakage, are deducted before estimating duties.

Tare is the deduction from the gross weight because of the weight of the box, etc. **Leakage** is an allowance made on liquids in casks or barrels.

5. The government by an Act of Congress, usually called The Tariff Act, determines the amount of duties to be paid on different kinds of merchandise.

6. The government designates certain places, called **ports of entry**, where merchandise may be brought into the country.

7. **Custom houses** are buildings built at the ports of entry, where the duties on the imported goods are paid.

8. Importers are required to submit to the collector an invoice. **An invoice** is a list of the goods, stating the quantity, and their cost in the country from which they are imported.

1. Invoices are made out in the money of the country from which the goods are imported. When changed to United States money, the duty is computed on the nearest dollar.

2. What is the duty at 28% on 20 casks of wine, containing 40 gal. each, invoiced at \$1.12½ a gallon, if 5% is deducted for leakage?

40 gal. × 20 = 800 gal.
5% of 800 gal. = 40 gal. leakage.
800 gal. − 40 gal. = 760 gal.
\$1.12½ × 760 = \$855. 28% of \$855 = \$239.40

3. What is the duty at 35% on 75 pieces of satin, each piece containing 47 yd., at \$1.65 a yard?

4. At 35% ad valorem, what is the duty on 150 doz. pairs of kid gloves invoiced at 68 francs a dozen? A franc = \$.193.

5. If the rate of duty is 50% and tare 2%, find the duty on merchandise invoiced at 4670 guilders. A guilder = \$.402.

6. What is the duty, at 30¢ a dozen and 15% ad valorem, on 750 doz. linen collars valued at 88½¢ a dozen?

7. Find the duty on 6 bbl. of varnish, 31½ gal. to a barrel, invoiced at \$6 a gallon, leakage 10%, if the duty is 35% ad valorem and \$1.32 a gallon.

8. At 44¢ a square yard and 40% ad valorem, find the duty on 2468 yd. Brussels carpet, ¾ yd. wide, invoiced at 3½ shillings. A shilling = \$.243.

9. Find the duty at \$.65 a cubic foot on a block of Italian marble, 2½ ft. by 3½ ft. by 8 ft.

10. The duty on 625 yd. of silk, at 40% ad valorem, is \$550. For how much a yard must the importer sell to gain 15%?

11. What is the duty on 46,080 pencils at 2¢ a gross?

12. How much is the duty at 1½¢ a pound on 36 boxes of figs, 24 lb. to a box, tare 4½ lb. to a box?

1. At 3¢ an ounce, what is the cost of 2 lb. of pepper?

2. What is the volume of a square pyramid whose altitude is 15 in., and each side of the base 10 in.?

3. A merchant lost 10% by selling flour at $4.50 a barrel. What did the flour cost the merchant?

4. By selling a chair for $6 a merchant gained 20%. What did the chair cost him?

5. What is the duty on 400 lb. of coffee at 4¢ a pound?

6. What is the duty at 25% on a bill of goods invoiced at $4000?

7. In example 6, is the duty specific or ad valorem? Which is it in example 5?

8. The list price of a bill of goods is $675. If the discount is $33\frac{1}{3}\%$, what is the net price?

9. A man's assets (what he owns) are $3000, and his liabilities (what he owes) are $4000. How much can he pay on a dollar?

10. A man's assets are $2400 and his liabilities $3600. How much can he pay on a dollar?

11. Find the number of board feet in a board 12 ft. long, 10 in. wide, and 1 in. thick.

12. How many feet long is the wall of a school-room which contains 264 sq. ft., if it is 12 ft. high?

13. How many acres are there in a lot 20 rd. by 16 rd.?

14. How do you find the net price of a bill of goods when two or more discounts are given?

15. When you know the value of imported goods and the rate of ad valorem duty, how do you find the total amount of duty?

16. When you know the assessed valuation of a man's property, and his tax, what can you find?

17. Find the ad valorem duty on goods invoiced at:

a. $500 at 30% *b.* $1200 at 25%

c. $600 at 40% *d.* $1200 at $33\frac{1}{3}\%$

1. A man spent $\frac{1}{3}$ of his money and then $\frac{3}{5}$ of the remainder. What part of his money did he spend? What part did he have left? If he spent \$91 more than he had left, how much had he at first?

2. A circular park is 60 rd. in diameter. At \$1.85 a rod, what will it cost to fence the park?

3. Ad valorem duty is levied on goods assessed at \$8250, making the total cost \$11,550. What is the rate of duty?

4. A man imported some goods on which he paid a specific duty of \$1925. If the duty was 22¢ a pound, how many pounds did he import?

5. If a man's property is assessed at \$9750, and his tax is \$117, what is the rate of taxation?

6. If a man takes out an insurance policy for \$5500, at \$28.40 per thousand, how much premium will he pay in 15 years?

7. A commission merchant sold 180 tubs of butter, weighing 50 lb. to the tub, at 24¢ a pound, charging 5% commission. Find the net proceeds of the butter.

8. If in a 30 ft. square you draw 4 circles, as large as possible, with their edges touching, how many square feet of space will not be included within the circles?

9. The base of a triangular lot is 24 ft. If the altitude is 45 ft., what is the area?

10. How many tiles, 4 in. by $2\frac{1}{2}$ in., will be required to make a hearth 6 ft. 4 in. long, and 3 ft. 9 in. wide?

11. A floor is 18 ft. by 10 ft. Find the number of yards of carpet, $\frac{3}{4}$ yd. wide, that will cover it in the most economical way.

12. A clock which loses 4 minutes a day is $\frac{1}{2}$ hour too fast at noon Sunday. At 6 P.M. of true time on Friday, what time will the clock indicate?

13. At the rate of \$160 an acre, find the value of a lot measuring 90.75 ft. in length and 34.5 ft. in width.

1. How many boxes, each holding $\frac{2}{3}$ of a quart, can be filled from 6 qt. of berries?

2. $\frac{5}{12}$ is what part of $\frac{2}{3}$?

3. 54 is $\frac{1}{8}$ more than what number?

4. If 21 sheep are worth \$49, what are 3 sheep worth? Do not find the cost of one sheep.

5. If 24 horses consume 50 bu. of oats in 5 days, how many bushels will 12 horses consume in the same time? How many bushels will 12 horses consume in a day?

6. .125 is what part of 1? .005 is what part of .01?

7. Change 8% to a common fraction in its lowest terms.

8. What is 4% of 80? What is $\frac{1}{5}$% of 75?

9. Find 20% of 135; of $\frac{5}{6}$; of .75.

10. 36 is 12% of what number? 25% of what number? 10% of what number? $33\frac{1}{3}$% of what number?

11. 60 miles is 25% of how many miles?

12. \$30 is 85% less than how many dollars?

13. When the cost is \$25 and the gain \$5, find the rate per cent of gain and the selling price.

14. An agent collected a debt of \$800, charging 3% commission. Find the commission.

15. An agent's commission on a sale was \$600. If the rate of commission was 3%, what was the amount of the sale?

16. On a bill of goods, amounting to \$40, a discount of 5% was allowed for cash payment. What was paid for the goods?

17. A mandolin is listed at \$15. If a music teacher is allowed a discount of 20%, find the price she must pay. If a further discount of 10% had been allowed, what would she then have paid?

18. Find the annual premium of a policy of \$10,000 at \$18.24 a thousand.

19. On a bill of \$100, which is the better discount, 40% and 10%, or 30% and 20%?

1. If the price of an organ is \$108 after discounts of 20% and 10% have been deducted, what is the list price?

2. On a bill of goods, amounting to \$1200 which is better for the purchaser, and how much better, 55% discount, or two successive discounts of 50% and 5%?

3. Find the net amount of a bill of \$1250, discounts being 25% and 4%. Find a single discount equivalent to these two discounts.

4. The whole tax of a town is \$16,020, and the taxable property is \$784,750. The number of polls is 260, each assessed \$1.25. If A's property is assessed at \$10,500, what is his tax?

5. A tax of \$673.50 was paid on property valued at \$44,900. What was the rate?

6. A building is insured for $\frac{4}{5}$ of its value at 1%. What is the value of the building if the premium is \$72.36?

7. A man takes out a \$5000 life insurance policy, payable in 20 years. If he pays an annual premium of \$45.50 a \$1000, how much will his insurance cost him if he lives till the policy falls due?

8. An agent returned to his employer as net proceeds of a sale, \$6165.75, after deducting \$93.75 for expenses, and a commission of $2\frac{1}{2}$%. What was the amount of the sale?

9. An agent received \$60 for selling potatoes at 50 cents a bushel on a commission of 4%. How many bushels did he sell?

10. A man drew out $33\frac{1}{3}$% of his bank deposit, and paid 25% of it for a house worth \$1800. What was his original bank deposit?

11. A man lost 20% of his money and then gained 10% on the sum he had left. If he then had \$352, how much had he at first?

12. If wheat produces 82% of its weight in flour, how much flour will be obtained from 280 bags of wheat, each weighing 180 lb.?

1. Smith & Murray sell lace curtains at \$10 a pair. If their gain is 25%, what did the curtains cost them?

2. A farmer sold a cow for \$40, which was 80% of the cost. What was his loss?

3. One merchant sold some dress goods at 60¢ a yard at a gain of 20%. The next day he advanced the price to 70¢ a yard. What was his gain per cent then?

4. A man bought a sleigh for \$35 and sold it so as to gain 20%. Find the selling price.

5. A man bought an automobile for \$1200 and sold it at a loss of $16\frac{2}{3}\%$. For what did he sell it?

6. At $\frac{1}{2}$ of 1% what is the discount on a bill of \$1000?

7. The gain on a certain article is 54 cents and the rate of gain 27%. Find the cost and selling price.

8. The cost of an article is 70¢ and the rate of gain $7\frac{1}{7}\%$. Find the gain and the selling price.

9. If Carrie's money is 25% less than her brother's, his money is what per cent of her money?

10. A merchant bought a bankrupt stock at 40¢ on the dollar and sold it at 20% below the original price. What per cent did he gain?

11. The rent of a house is \$360. If this is 12% of the value of the house, what is its value?

12. A piece of cloth containing 12 yd. was sold for \$60 at a gain of 25%. What was the gain on each yard?

13. Four square feet is what per cent of a 4 ft. square?

14. What number, increased by 20% of itself, will equal 120?

15. What number, decreased by 20% of itself, will equal 160?

16. What number, increased by 10% of itself, will equal 121?

17. What number, decreased by 10% of itself, will equal 270?

1. A grocer bought 150 boxes of oranges at \$2.50 a box. He found 16% spoiled, but sold the rest at \$3.50 a box. How much did he gain or lose?

2. A merchant sells his goods at an average profit of 15%. What amount of goods must he sell during the year to have an annual gain of \$13,500?

3. In selling an article the manufacturer made a profit of 25%, the wholesale dealer a profit of 12%, and the retail dealer a profit of 15%. If the customer paid \$6.44 for the article, what was the cost of manufacture?

4. A lady paid \$27 for a cloak. $66\frac{2}{3}\%$ of the cost of the cloak was 90% of the sum paid for other things. How much did she pay out in all?

5. A man paid \$5000 for land and then spent a sum equal to 80% of this amount for a new house. If he should then sell the whole for \$12,000, what per cent would he make?

6. An agent collected $87\frac{1}{2}\%$ of a bill of \$2400 and charged 5%. How much should he remit to his employer?

7. A man owned 600 acres of land. He sold 25% of it to one man and $33\frac{1}{3}\%$ to another. At \$75 an acre find the value of the part not yet sold.

8. A real estate dealer bought 80 acres of land for \$3000 and sold it at a profit of \$7.50 an acre. What was the gain per cent?

9. A farmer asked \$40 for a cow and refused an offer of 20% less than he asked. He sold her for 25% above the offer that he received. At what price did he sell?

10. When a clerk received \$100 a month salary, he paid \$800 a year for his living expenses, and saved the rest. When his salary was increased 25%, he increased his expenses 30%. How much more or less did he save than before his increase?

11. What is the value of 75% of a farm, if $62\frac{1}{2}\%$ of it is worth \$4000?

Find the interest on:

1. \$800 from July 15, 1910, to Jan. 1, 1914, at 6%.
2. \$1200 from Aug. 16, 1910, to Feb. 10, 1913, at 5%.
3. \$900 from Sept. 17, 1911, to March 13, 1912, at 4½%.
4. \$625.50 from Oct. 20, 1910, to April 12, 1914, at 6%.
5. \$819.40 from Nov. 18, 1911, to May 20, 1915, at 4%.
6. \$512.80 from Dec. 11, 1910, to June 21, 1913, at 4½%.
7. \$419.60 from Jan. 3, 1912, to July 15, 1915, at 5%.
8. \$718.40 from Feb. 6, 1912, to Aug. 27, 1914, at 5½%.
9. \$361.10 from March 13, 1911, to Sept. 28, 1913, at 6%.
10. \$985.50 from April 4, 1911, to Oct. 19, 1914, at 6%.
11. \$895.90 from May 27, 1912, to Nov. 3, 1915, at 5%.
12. \$585.80 from June 20, 1911, to Feb. 1, 1914, at 4½%.
13. \$436.20 from July 16, 1912, to March 3, 1915, at 4%.
14. \$664.30 from Aug. 11, 1911, to April 2, 1915, at 3%.
15. \$708.70 from Sept. 18, 1912, to May 10, 1916, at 4%.
16. \$610.20 from Oct. 3, 1911, to June 5, 1914, at 4½%.
17. \$720.30 from Nov. 21, 1910, to July 1, 1913, at 5%.
18. \$830.40 from Dec. 16, 1910, to Aug. 3, 1915, at 6%.
19. \$940.50 from July 25, 1911, to Sept. 7, 1914, at 6%.
20. \$46,721 from June 13, 1912, to Oct. 2, 1915, at 5½%.
21. \$35,610 from Aug. 19, 1911, to Nov. 4, 1914, at 4%.
22. \$24,509 from May 17, 1911, to Dec. 11, 1913, at 4½%.
23. \$13,498 from Sept. 9, 1912, to May 7, 1915, at 5%.
24. \$62,597 from April 20, 1912, to June 10, 1914, at 6%.
25. \$51,486 from Oct. 17, 1911, to April 3, 1913, at 4½%.
26. \$40,375 from March 24, 1912, to July 5, 1915, at 4%.
27. \$39,264 from Nov. 20, 1911, to March 10, 1914, at 6%.
28. \$28,153 from Feb. 10, 1912, to August 4, 1914, at 5½%.
29. \$17,042 from Dec. 8, 1910, to Feb. 26, 1913, at 6%.
30. \$86,931 from Jan. 7, 1912, to Sept. 13, 1914, at 6%.

1. A **corporation** is a company composed of several persons, authorized by law to carry on a certain kind of business.

2. Banks, railroads, insurance companies, mining companies, and many manufacturing companies are organized as stock companies or corporations.

3. The **charter** is the certificate given to the corporation, usually by the legislature of the state, stating the name, object, amount of capital, etc., of the corporation.

4. The **capital** or **capital stock** is the amount of money and other property owned by the corporation.

5. A **share** is one of the equal parts into which the stock of a company is divided. The face value of a share is usually $100, and will be so considered in this book unless stated to the contrary.

6. Shares are offered for sale and bought by people who think the business is likely to be a good investment.

7. A **stockholder** is a person who owns one or more of the shares of the corporation.

8. **Certificates of stock** are given to every stockholder, stating the number of shares he owns and the **face or par value** of each certificate.

9. After receiving the charter, the stockholders elect such officers as president, secretary, treasurer, and board of directors, who have the entire management of the business.

10. A stockholder cannot demand of the corporation the return of his money, but he may sell his shares.

11. Stock is said to be **at par** when it sells for its face value. If the company is prosperous, the shares usually sell for more than their face value, that is, **above par** or **at a premium**. If the company is not prosperous, the shares will sell for less than their face value, that is, **below par** or at a **discount**.

12. **Dividends** are the profits of a company after all expenses are paid. These dividends are divided among the stockholders according to the number of shares each holds.

1. When a business is not successful, the stockholders may be called upon to make up the loss by an **assessment.**

2. The dividends or assessments of a stockholder do not depend on the price at which the shares were bought, but are reckoned on their par value.

3. Shares of stock are bought and sold in the market like other property.

4. **Brokers** are persons who make a business of buying and selling stocks and bonds.

5. Brokers are really commission agents and charge a commission called **brokerage.** It is always reckoned on the par value of the stock.

6. Corporations issue two kinds of stocks, preferred and common. Owners of preferred stock usually receive a stated dividend before the owners of common stock receive anything.

7. When corporations, or national, state, or city governments, borrow large sums of money, they give **bonds,** that is, interest-bearing promissory notes.

8. **Coupons** are certificates of interest attached to the bonds. These coupons are cut off and presented at a bank for payment.

9. Bonds are usually named according to their rate of interest and date of maturity.

Mass. 5's, 1915, at 103 means Massachusetts State bonds, paying 5% interest, due in 1915, and selling at 103% of its par value. 700 Union Pacific, 95, means 700 shares of Union Pacific stock at 95% of its par value.

10. Bonds bear interest at a stated rate. The rate of dividend declared on stocks depends upon the prosperity of the corporation.

11. Records of stock sales, called **quotations,** may be found in most newspapers. Read the following, taken from the quotations of the Boston Stock Exchange:

Mich. Tel. & Tel. 5's, 97 100 Boston & Lowell, 105

New York Stock Exchange.
4300 Bklyn. Rap. Tr. $121\frac{1}{4}$.
4900 Can. Pac. $197\frac{3}{4}$.
3100 Rock Island $43\frac{1}{4}$.
2800 Union Pac. $184\frac{1}{2}$.
300 West. Un. Tel. $67\frac{1}{2}$.
100 National Tradesmens 185.
50 Adams Express 248.
500 N. Y., N. H. & H. R. R. 155.

1. Which of these stocks are selling above par? Which below par? Which at a premium? Which at a discount?

2. What is the cost of 320 Western Union Telegraph shares as quoted above, brokerage $\frac{1}{8}\%$?

$\$67\frac{1}{2} + \$\frac{1}{8} = \$67\frac{5}{8}$, total cost of 1 share.
$\$67\frac{5}{8} \times 320 = \$21{,}640$.

Brokerage is always reckoned on the par value. $\frac{1}{8}\%$ of \$100 = $\$\frac{1}{8}$. When stock is quoted at $\$67\frac{1}{2}$, one share will cost the purchaser $\$67\frac{5}{8}$.

3. A man sold through a broker at $\frac{1}{8}\%$, 240 shares of Canadian Pacific. How much did he receive?

$\$197\frac{3}{4} - \$\frac{1}{8} = \$197\frac{5}{8}$.
$\$197\frac{5}{8} \times 240 = \$47{,}430$.

In selling the owner receives the market price less the brokerage.

4. How much must be paid for 400 shares of Brooklyn Rapid Transit as quoted above, brokerage $\frac{1}{8}\%$?

5. How much would a man receive for 50 shares of National Tradesmens stock, sold at price quoted, after paying a brokerage of $\frac{1}{8}\%$?

6. A man bought stock at 15% below par and sold it at 10% above par, brokerage in each case $\frac{1}{8}\%$. How much did he make on 115 shares?

7. Including $\frac{1}{8}\%$ brokerage, what is the cost of 23 shares of stock, bought at $8\frac{1}{2}\%$ discount?

8. Find the cost of 325 shares of B. & A. stock selling at $184\frac{1}{2}$, brokerage $\frac{1}{8}\%$.

9. 148 shares of the stock of the Pullman Company sold yesterday at 159. Find the amount received, brokerage $\frac{1}{8}\%$.

1. How many shares of bonds at $97\frac{3}{8}$ can be bought for \$1462.50, brokerage $\frac{1}{8}\%$?

$\$97\frac{3}{8} + \$\frac{1}{8} = \$97\frac{4}{8}$.

$\$1462.50 \div \$97.50 = 15$.

2. How many shares at $110\frac{3}{8}$ can be bought for \$12,265.50, brokerage $\frac{1}{8}\%$?

3. A man bought some railroad stock at $59\frac{7}{8}\%$ premium, brokerage $\frac{1}{8}\%$, paying \$19,000. How many shares did he buy?

4. What is the income from 25 shares of stock bought at 110, if it yields an annual dividend of 6%?

$\$100 \times 25 = \$2500$ par value.
6% of \$2500 = \$150.

Dividends are always paid on the par value of the stock.

5. A man invests \$36,000 in bank stock at $89\frac{7}{8}$, brokerage $\frac{1}{8}\%$. It yields $3\frac{1}{2}\%$ semi-annual dividends. Find his annual income.

NOTE: First find the number of shares bought as in example 1. Then find the par value and the income as in example 4.

6. What income would a man receive from \$19,971 invested in U. S. 4's, bought at $118\frac{3}{4}$, brokerage $\frac{1}{8}\%$?

7. What annual income will a man receive from investing \$5765.50 in 6% stock bought at $110\frac{3}{4}$, brokerage $\frac{1}{8}\%$?

8. A man paid \$21,978 for R. R. 6's at $110\frac{7}{8}$, brokerage $\frac{1}{8}\%$. What income should he receive?

9. Mr. Smith owned a farm of 120 acres, which yielded him an annual income of $\$2.62\frac{1}{2}$ an acre. A real estate agent sold the farm for \$75 an acre, charging him 3% commission. Mr. S. invested the net proceeds in $3\frac{1}{2}\%$ R. R. stock at $87\frac{7}{10}$, brokerage $\frac{1}{8}\%$. How much did he increase his income?

10. A man invested \$6804 in 5% bonds bought at $5\frac{5}{8}\%$ discount, brokerage $\frac{1}{8}\%$. What is his annual income from the bonds?

11. A man invested \$76,800 in $2\frac{3}{4}\%$ bonds at $95\frac{7}{8}$. How much stock at $109\frac{7}{8}$ can he buy with his first annual dividend, $\frac{1}{8}\%$ brokerage in each transaction?

1. How much 4% stock must be bought to give an income of $1280?

$1280 ÷ $4 = 320 shares. Divide the given income by the income on 1 share to find the number of shares.

2. When U. S. 4's are bought at 120, what is the rate of income?

$4 ÷ $120 = .03$\frac{1}{3}$ = 3$\frac{1}{3}$%. Divide the income on 1 bond by the cost of the bond to find the rate of income.

3. A man's income from bonds is $1428. What amount did he invest in bonds paying 4$\frac{1}{4}$%, at 103$\frac{3}{8}$, brokerage $\frac{1}{8}$%?

$1428 ÷ $4.25 = 336, no. of bonds.
103\frac{1}{2}$ × 336 = $34,776, cost.

4. What is the rate of income, when 7% stock is bought at 139$\frac{7}{8}$, brokerage $\frac{1}{8}$%?

5. What sum must be invested in stock at 112, brokerage $\frac{1}{8}$%, which pays 10% annually, to obtain an income of $5500?

6. Which will give you the larger income, 7% stock at 108 or 6% stock bought at 105?

7. What is the price of stock when $8729 will purchase $11,600 worth of stock?

8. A man bought 75 shares of bank stock at 108$\frac{1}{2}$. After receiving a dividend of 6%, he sold the stock for 107. How much did he gain, allowing $\frac{1}{8}$% brokerage for buying and for selling?

9. A man who held $8000 worth of 3$\frac{1}{2}$% stock wished to increase his income. He sold the stock at 87$\frac{1}{2}$ and invested the proceeds in 5% stock at 116$\frac{2}{3}$. How much change did he make in his income if no brokerage was paid in either case?

10. If a man receives $318.75 as a dividend on $5000 worth of stock, what is the per cent of the dividend?

11. What sum must I invest in Massachusetts 5's, purchased at 97$\frac{1}{2}$, brokerage $\frac{1}{8}$%, to receive an annual income of $1200?

1. How much stock will \$6400 buy at 80?

2. How much stock will \$37,500 buy at 75?

3. What premium must be paid for insuring a cargo of goods for \$16,000 at $\frac{7}{8}\%$?

4. A typewriter was bought for \$120 and sold for \$100. What per cent was lost? What per cent would have been gained if it had been sold for \$150?

5. How do you change any given per cent to the form of a common fraction?

6. When you know what a certain per cent of a number is, what can you find? How will you find it?

7. How do you find the interest on any sum of money for 60 days at 6%? How would you find it for 90 days?

8. When you know the assessed valuation of a man's property, and the rate of taxation, what can you find?

9. Find the cost of 100 shares of stocks at $113\frac{7}{8}$, brokerage $\frac{1}{8}\%$.

10. Find the income from the shares in example 9, if the rate of dividend is 8%.

11. Can you read the stock quotations in the newspapers? Read 25 W. End St. Ry. pr. $98\frac{1}{2}$, and tell what it means.

12. If you should sell some bank stock through a broker at market price and buy it back at the same market price, how much would you lose on a share?

13. State the four steps necessary to take to find the proceeds on any given non-interest bearing note.

14. A deposited a certain sum of money in a savings bank and left it undisturbed for 3 years at 4%. B loaned a friend the same sum of money at the same rate. At the end of 3 years which man will receive the larger amount of interest?

15. How many board feet are there in 40 boards, each 6 in. wide and 10 ft. long?

16. If 8% of my money is \$24, how much money have I?

17. If 4% of my money is \$32, how much money have I?

1. Find the duty at 42¢ a square yard, and 35% ad valorem, on 1836 yd. of carpet $\frac{3}{4}$ yd. wide, invoiced at 6¢ a yard.

2. A capitalist invested \$27,600 in stock at $114\frac{7}{8}$, brokerage $\frac{1}{8}$%. The company declared a dividend of $7\frac{1}{2}$%. What was his share of the dividend?

3. Mr. Avery sold 560 shares of stock, paying 4%, at 80, without brokerage, and loaned the proceeds at 6%. How much was his income changed?

4. A man paid \$72 for insuring property worth \$12,000 at the rate of 90¢ on every \$100. What part of the property was insured?

5. What is the ratio of the entire surface of a cylinder having an altitude of 12 in., and a diameter of 12 in., to the surface of a sphere 12 in. in diameter?

6. What is the ratio of the volume of the cylinder in example 5 to the sphere?

7. A tax collector received \$350.36 as his 2% commission on taxes collected. The assessed valuation of the taxable property was \$1,844,000. What tax must a man pay whose property was valued at \$7500?

8. A man cuts a 2′ strip of grass entirely round a rectangular lawn 50′ × 150′. How many square feet of grass does he cut?

9. He then cuts another strip, 2′ wide, round the lawn. How many square feet of grass are cut the second time round the lawn?

10. The weights of five bags of timothy seed are as follows: 105 lb., 104 lb., 106 lb., 107 lb., 110 lb. Allowing 1 lb. each for the weight of the bags, and 45 lb. as the weight of a bushel, how much is the seed worth at \$1.85 a bushel?

11. Find the amount of \$175 at 5% for 4 yr. 9 mo. 15 da.

12. A piece of land, 35 rd. long and 7 rd. wide, is divided into 5 square lots of equal size. What will be the cost of boundary and cross fences at \$2.12$\frac{1}{2}$ a rod?

1. Mr. Jones and Mr. Davis decided to carry on a clothing business together under the firm name of Jones and Davis, each putting in $5000.

2. A **partnership** is an association of two or more men in business. The association is called a **firm** or **company** or **house**. The persons associated are called **partners**.

3. The capital of the firm is the sum of all the money invested by each partner. Each partner is supposed to contribute something toward the firm, as money, goods, labor, skill, or good will.

4. The gains and losses of a firm are shared in proportion to the amount of capital invested and the time of their investment. Partnership is then only one form of proportion.

5. A, B, and C formed a partnership. A invested $6000, B $8000, and C $6000. The profits for a year were $6000. Find each partner's share of the profits.

$6000 + $8000 + $6000 = $20,000, the whole capital.
$\frac{6000}{20000} = \frac{3}{10}$ = A's share of the capital.
$\frac{3}{10}$ of $6000 = $1800, A's share of the gain.
Find B's and C's shares in the same way.

6. A and B entered into partnership. A furnished $3000 for 2 mo., and B $2000 for 6 mo. If they gained $1500, what was each one's share of the profits?

$3000 \times 2 = \$\ 6000$
$2000 \times 6 = \$12000$
$\$18000$

$3000 in business for 2 mo. is the same as $6000 in business for 1 mo., and $2000 in business for 6 mo. is the same as $12,000 for 1 mo. $6000 and $12,000 should now be considered as the respective capitals. Then proceed as in example 5.

NOTE: In this example the profit of each partner depends on two elements, the amount of his capital and the time it is employed. The element of time must be eliminated by finding the equivalent capital for 1 mo., before the principles of partnership can be applied.

7. A, B, and C formed a partnership. A put in $2000 for 10 mo., B $1800 for 8 mo., and C $3000 for 6 mo. If they gained $2620, what was each partner's share?

1. A and B formed a partnership. A invested \$3000, and B \$1000. How should they divide a gain of \$1200?

2. Charles and Henry formed a partnership. Charles invested \$15, and Henry \$10. How ought a gain of \$30 to be divided between them?

3. Two boys invested \$14 in business. If one invested \$4 to the other's \$3, how much did each invest?

4. Jan. 1, 1911, A and B engaged in business, each contributing \$6000. April 1, 1911, C invested \$8000, and Oct. 1, 1911, D invested \$9000. The gain at the end of the year was \$14,580. Find the share of each.

5. A and B engaged in business Jan. 1, 1911. A invested \$5000, and B \$8000. On Aug. 1 they took in C as a third partner, with a capital of \$7000. On Jan. 1, 1912, their net gain was \$7640. What was the share of each partner?

6. D commenced business with \$1800 worth of stock. Two months later he took in E with a capital of \$1500. A month later F was admitted with a capital of \$2400. At the end of the year the firm had gained \$1164. Find the share of each.

7. A, B, and C are partners with a capital of \$4040. A's gain for the year is \$492, B's \$450, and C's \$270. What capital did each furnish?

8. Two farmers pasture cattle in a field. The first pastures 48 head for 12 weeks, and the second 72 head for 8 weeks. If the bill was \$172.80, how much should each pay?

9. Three men delivered milk at a creamery. The first delivered 1552 lbs., the second 1876 lbs., and the third 1740 lbs. If they received \$77.52, what was the share of each?

10. Two men hire a pasture for \$45. One puts in 21 cows, and the other 24 cows. What should each pay?

11. A, B, and C formed a partnership. A invested \$5000 for 8 months, B \$6400 for 10 months, and C \$4000 for 12 months. Find each partner's share of a gain of \$3705.

1. Anna and Mary have 21 cents. If Anna has 2 times as many cents as Mary, how many cents has Mary?

Let x = number of cents Mary has.
$2x$ = number of cents Anna has.
$3x$ = 21 cents that both have.
x = 7 cents that Mary has.

In examples like this we let the symbol x stand for the unknown quantity that we need to know. Why do we say $2x$ = the number of cents that Anna has? Where do we obtain $3x$ = 21 cents? If $3x$ = 21 cents, then x = $\frac{1}{3}$ of 21 cents, or 7 cents.

2. An **equation** is a statement that two expressions stand for the same number. In example one, $3x = 21¢$ is an equation. $3x$ and 21 both stand for the total number of cents.

3. Finding the value of x, or the unknown number, in any equation is called **solving the equation.**

4. Every equation consists of two parts called members. In the equation above, $3x$ is the **first member**, and 21 is the **second member.**

5. In algebra we have two kinds of numbers, positive and negative. The sign + is prefixed to positive quantities, and the sign − to negative quantities. The expression $-6x$ may be read *negative* $6x$, and the expression $+6x$ may be read *positive* $6x$. When neither sign precedes any term, the term is positive.

6. Negative numbers are as real as positive numbers and compared with positive quantities mean opposite in direction or effect.

7. If we call gain in business positive, what shall we call the loss?

8. If 15° below zero is called negative, what should 15° above zero be called?

9. If direction to the east is called positive, what should direction to the west be called?

10. Change the following expressions, using positive terms for negative, and negative terms for positive:

a. − 4° south latitude. *b.* I gained + $5.
c. I lost − $2. *d.* + 5° west longitude.

$-6, -5, -4, -3, -2, -1, 0, +1, +2, +3, +4, +5, +6.$

$-$ ← → $+$

1.	2.	3.	4.	5.
$+4$	-4	$+4$	-4	-2
$+2$	-2	-2	$+2$	$+4$
$+6$	-6	$+2$	-2	$+2$

The 5 examples above may be illustrated as follows by using the diagram at the top:

1. Put your pencil on $+4$ and move it in the positive direction two places. You are now at $+6$.

2. Put your pencil on -4 and move it in the negative direction two places. You are now at -6.

3. Put your pencil on $+4$ and move it in the negative direction two places. You are now at $+2$.

4. Put your pencil on -4 and move it in the positive direction two places. You are now at -2.

5. Put your pencil on -2 and move it in the positive direction four places. You are now at $+2$.

From the illustrations we see that:

a. When the signs are alike, we add the terms, keeping the same sign.

b. When the signs are unlike, we subtract the terms and give to the difference the sign of the greater term.

6. Add: $14a, -3a, 8a, -2a, -4a, 7a, -a, 18a$.

Combine the positive quantities, and then the negative, before finding the algebraic sum.

7.	8.
$2ac + 4xy$	$3a - 2b + 6c$
$5ac - 2xy$	$2a + 3b - 5c$
$3ac + 2xy$	$4a - b + 2c$
$-2ac + 3xy$	$-5a - 4b - 3c$

9. $4b + 3c, -3b - 5c, -2b - 5c, -6b + c.$

10. $6a - 5c + b, -5a - 4c - 5b, 2a + c - b.$

11. $5ab - 8a^2b + x^2y + xy^2, 4a^2b - 4x^2y - 7ab + 5xy^2, 3ab + 4a^2b - 3x^2y + 4xy^2.$

1. There are two numbers one of which is twice the other, and their sum is 45. What are the numbers?

2. What number is that to which if six be added, the sum will be 15?

3. Divide 23 cents between John and James, giving to James 5 more than to John.

4. The sum of two numbers is 25, and the smaller is 7 less than the larger. What are the numbers?

5. Robert lost 12 cents and has 17 cents left. How many cents had he at first?

6. If $\frac{3x}{4} = 6$, what will $\frac{x}{4}$ equal?

7. If each member of the equation $\frac{3x}{4} = 6$ is multiplied by 4, what equation will express the product?

8. Divide each member of the equation $4x = 12$ by 4. Express the quotient as an equation.

9. The sum of two numbers is 18, and one is half as large as the other. What are the numbers?

Let $2x$ = the larger number and avoid a fraction.

10. A boy sold a knife for 30 cents and thus gained $\frac{1}{4}$ of the cost. How much did it cost?

11. Charles spent $\frac{1}{3}$ of his money and had 6 cents left. How many cents had he at first?

12. The difference between Samuel's age and $\frac{2}{5}$ of his age is 15 years. What is his age?

Find the value of x:

13. $x - 7 = 4$.
14. $5x + 2 = 12$.
15. $2x + 6 = 8$.
16. $4x + 5 = 13$.
17. $3x - 7 = 11$.
18. $2x - 9 = 21$.
19. $2x - 5 = 7$.
20. $3x - 7 = 20$.

21. The product of a and b is not written $a \times b$, but ab. The product of 5, x, and y is written $5xy$.

22. Write the product of 8 and c; of 4, a, and d; of 7, x, y, and z.

1. You learned last year that the equality of an equation is not changed when:

a. The same number is added to both members.
b. The same number is subtracted from both members.
c. Both members are multiplied by the same number.
d. Both members are divided by the same number.

a.
$$\begin{aligned} x - 4 &= 5 \\ 4 &= 4 \\ \hline x &= 9. \end{aligned}$$

By adding 4 to both members we get rid of the negative 4 in the first member.

b.
$$\begin{aligned} x + 2 &= 6 \\ 2 &= 2 \\ \hline x &= 4. \end{aligned}$$

By subtracting 2 from both members we get rid of the positive 2 in the first member.

c.
$$\begin{aligned} \tfrac{1}{3}x &= 4 \\ x &= 12 \end{aligned}$$

In this equation we wish to get rid of the fraction, so we multiply both members by 3. $3 \times \frac{1}{3}x = x$. $3 \times 4 = 12$.

d.
$$\begin{aligned} 5x &= 20 \\ x &= 4. \end{aligned}$$

We know the value of $5x$. To find the value of x, we divide both members by 5.

Find the value of x in the following:

2. $x + 9 = 13$.	3. $x - 5 = 7$.	4. $\frac{1}{4}x = 4$.
5. $3x = 18$.	6. $x + 3 = 8$.	7. $x - 1 = 10$.
8. $\frac{1}{5}x = 2$.	9. $9x = 27$.	10. $x + 7 = 9$.
11. $x - 8 = 3$.	12. $\frac{1}{2}x = 13$.	13. $2x = 40$.
14. $2x + 4 = 12$.	15. $3x - 2 = 7$.	16. $\frac{3}{5}x = 6$.
17. $4x + 5 = 25$.	18. $4x - 3 = 13$.	19. $\frac{5}{6}x = 10$.
20. $5x - 6 = 14$.	21. $3x + 3 = 18$.	22. $\frac{2x}{3} = 4$.
23. $7x - 5 = 9$.	24. $8x + 4 = 20$.	25. $\frac{3x}{4} = 12$.
26. $3x + 4 = 19$.	27. $5x - 4 = 21$.	28. $12x = 36$.
29. $7x - 2 = 40$.	30. $3x + 4 = 22$.	31. $\frac{1}{2}x = 3$.
32. $2x + 3 = 17$.	33. $7x - 5 = 23$.	34. $\frac{3x}{7} = 3$.
35. $3x + 7 = 43$.	36. $2x - 9 = 15$.	37. $\frac{3x}{4} = 9$.
38. $\frac{3x}{5} = 12$.	39. $\frac{2x}{3} = 10$.	40. $7x = 21$.

1. $2x + 7 = 15$
$2x = 15 - 7$
$2x = 8$
$x = 4.$

You learned on page 215 how to get rid of the positive 7 in the first member. We see that subtracting 7 from both members is the same as transposing the $+ 7$ to the other member with the opposite sign.

2. $2x - 4 = 6$
$2x = 6 + 4$
$2x = 10$
$x = 5.$

In this example we see that adding 4 to both members is the same as transposing $- 4$ to the other member with the opposite sign.

Learn: Any term may be transposed from one member of an equation to the other member provided its sign is changed.

3. $3x - 18 = 7 - 2x$
$3x + 2x = 7 + 18$
$5x = 25$
$x = 5.$

The first step in solving an equation is to get all the known terms into one member and all the unknown terms into the other member. We have just learned that we can do this by transposing, so we take $- 2x$ from the second member and place it in the first member as $+ 2x$, and we take $- 18$ from the first member and place it in the second member as $+ 18$. The equation is now $3x + 2x = 7 + 18$. The next step is to *unite the terms*. Uniting we have $5x = 25$.

Find the value of x in the following:

4. $8x + 12 = 2x + 18.$ **5.** $7x - 13 = 7 - 5x.$

6. $16x + 14 = 8x + 54.$ **7.** $8x - 15 = 6x - 3.$

8. $9x + 12 = 3x + 30.$ **9.** $5x - 4 = 6 + 3x.$

10. $\frac{x}{2} - 8 = \frac{x}{4} - 4$
$\frac{x}{2} - \frac{x}{4} = 8 - 4$
$2x - x = 16$
$x = 16.$

We first transpose as in example 3. We next multiply by 4, the least common multiple of the denominators, giving us the equation $2x - x = 16$. This process is called **clearing the equation of fractions.**

Find the value of x in the following:

11. $\frac{x}{3} + 4 = \frac{2x}{4} + 3$ **12.** $\frac{2x}{4} - 2 = \frac{x}{3}$

13. $\frac{3x}{9} + 7 = \frac{4x}{6} + 4$ **14.** $\frac{7x}{3} - 5 = \frac{5x}{3} - 3$

Learn: To clear an equation of fractions multiply each term of both members by the least common multiple of the denominators.

Find the value of x in the following:

1. $2x + \frac{6x}{4} - \frac{2x}{4} = 24$

2. $\frac{6x}{4} - \frac{10x}{8} = 44$

3. $\frac{7x}{2} + \frac{x}{4} + \frac{x}{8} = \frac{31}{2}$

4. $\frac{x}{4} - \frac{33}{2} = \frac{35}{2} - \frac{x}{4}$

5. $\frac{3x}{4} - 7 = \frac{2x}{3} + 8$

6. $2x + \frac{5x}{6} - 8 = 9$

7. $\frac{x}{2} - 3 + \frac{x}{3} = 5 - 3$

8. $\frac{x}{4} + \frac{x}{8} - \frac{x}{6} = \frac{5}{12}$

9. $\frac{2x}{3} + 12 = \frac{4x}{5} + 6$

10. $3x + \frac{3x}{4} + 15 = \frac{x}{2} + 41$

11. Edith is 6 years more than $\frac{1}{2}$ the age of her brother. If their united ages amount to 42 years, how old is each?

Let x = the brother's age
then $\frac{1}{2}x + 6$ = Edith's age
and $x + \frac{1}{2}x + 6 = 42$.
$2x + x + 12 = 84$
$2x + x = 72$
$3x = 72 \quad x = 24$
$\frac{1}{2}x + 6 = 18$.

Each problem in algebra requires its own statement or equation. Since each differs from any other no definite rule can be given. Practice will give you skill in translating the words of the problem into algebraic equations used in the solution.

12. Three boys have together 160 marbles. The first has 50 less than the second and 40 more than the third. How many has each?

Let x = the number the first has
then $x + 50$ = the number the second has
and $x - 40$ = the number the third has
$3x + 50 - 40 = 160$
$3x = 150 \quad x = 150$.

The statement is made that the first has 50 less than the second. To make our equation for the second, it is necessary for us to turn the statement so that it reads, the second has 50 more than the first. Make a statement for the third. Study the problem until you can tell where each equation given in the solution comes from.

1. Divide 164 into 3 parts, such that the first shall be 12 greater than the second, and the second 16 greater than the third.

2. What number, multiplied by 8 and then diminished by 13, is equal to 11?

3. The sum of 3 numbers is 140. The second is 4 times the first, and the third is $\frac{1}{2}$ of the second. What are the numbers?

4. A farmer sold $\frac{2}{5}$ of his farm to one man, $\frac{1}{3}$ of it to another, and had 20 acres left. How large was his farm at first?

5. If to a certain number, itself, $\frac{1}{5}$ of itself, and 5 be added, the sum will be 104. What is the number?

6. Three eighths of a certain number is 30 less than the number itself. What is the number?

7. A piece of land was sold for \$860 at a gain of 25%. What was the cost?

NOTE: Call the gain $\frac{25x}{100}$ or $\frac{x}{4}$.

8. When the selling price of an article is \$428.50 and the rate of loss 15%, what is the cost?

9. Damaged goods were sold for \$450, which was 10% below cost. Find the cost.

10. A man sold some property for \$2560, which was 20% below cost. What was the cost?

11. Divide 36 into three parts so that the first will equal $\frac{1}{2}$ of the third, and the second will equal the sum of the first and third.

12. $\frac{1}{4}$ of a given number added to 2 equals $\frac{3}{8}$ of itself plus 1. Find the number.

13. George has 12 more than $\frac{1}{2}$ as many cents as his sister Mary. If together they have \$1.74, how many cents has each?

14. George and William have together 36 marbles. George's number is to William's as 9 to 3. How many has each?

1. Robert and John together have eleven cents. If Robert has 2 more than $\frac{1}{2}$ as many as John, how many has each?

2. Find the value of x in the equation $x + 7 + \frac{2x}{5} + 3 = 24$.

3. A man is 17 years older than his sister. If his age is added to $\frac{3}{8}$ of his sister's age, the sum will be 61. What is the age of each?

4. A boy is 4 times as old as his playmate, and the difference of their ages is 12 years. What is the age of each?

5. A father is 5 times as old as his son, and the difference of their ages is 32. What is the age of each?

6. If $\frac{2}{3}$ of a number and 7 more be added to $\frac{1}{6}$ of the same number, the sum will be equal to the number. What is the number?

7. If from a certain number $\frac{1}{4}$ and $\frac{1}{6}$ of the number be subtracted, the remainder will be 14. What is the number?

8. The difference between $\frac{2}{3}$ and $\frac{4}{7}$ of a certain number is 4. What is the number?

9. $\frac{2}{5}$ of a pole are in the water, $\frac{1}{10}$ in the mud, and the remainder out of water. If there are 9 ft. more of it in the water than in the mud, how long is the pole?

10. A and B gained a certain sum of money in trade, of which A received \$10 less than B. If A's share is subtracted from twice B's, the remainder will be \$57. How much money did each receive?

11. If a number is added twice to itself, the sum will be 27. What is the number?

12. Divide 36 into two parts, one of which shall be 12 less than twice the other.

13. The length of a rectangular lot is 20 rd. more than the breadth. If the perimeter is 200 rd., how many acres are there in the field?

14. Three fourths of a number is 3 less than the number itself. What is the number?

1. What is the product of the two equal factors 4 and 4?

2. What is the product of the three equal factors 4, 4, and 4?

3. A **power** is the product of two or more equal factors. The product of two equal factors is the **second power** or **square**. The product of three equal factors is the **third power** or **cube**, etc.

4. What is the second power of 3? What is the third power? The fourth power? The fifth power?

5. A **root** is one of the equal factors, used in producing the power. If there are two equal factors, each is a **second root** or **square root**. If there are three equal factors, each is a **third root** or **cube root**.

6. 4^2 is read "the second power of 4" or "the square of 4." 4^3 is read "the third power of 4" or "the cube of 4."

7. The **exponent** is the figure written at the right and a little above the root. It indicates the desired power.

8. A root is indicated by $\sqrt{\ }$, which is called the **radical sign**. If any other root than the square root is desired, it is shown by the figure, called the **index**, which is placed above the sign. Thus, $\sqrt[3]{27}$ indicates the cube root of 27, which is 3.

Find the powers as indicated:

9. 2^3	10. 4^2	11. 5^3	12. 12^2	13. 10^4
14. 4^3	15. 1^5	16. 3^4	17. 2^5	18. 5^2
19. 54^3	20. 67^2	21. 1.42^3	22. 1.01^4	23. 37^3
24. $.01^4$	25. $.22^3$	26. $.14^2$	27. $.001^5$	28. 36^3
29. $(\frac{3}{4})^2$	30. $(\frac{1}{2})^3$	31. $(2\frac{1}{2})^3$	32. $(3\frac{1}{3})^3$	33. $(4\frac{1}{4})^2$

Find the roots as indicated:

34. $\sqrt{36}$	35. $\sqrt{25}$	36. $\sqrt{81}$	37. $\sqrt{16}$	38. $\sqrt{49}$
39. $\sqrt{4}$	40. $\sqrt{9}$	41. $\sqrt{64}$	42. $\sqrt{100}$	43. $\sqrt{144}$
44. $\sqrt[3]{8}$	45. $\sqrt[3]{27}$	46. $\sqrt[3]{64}$	47. $\sqrt[3]{125}$	48. $\sqrt[3]{216}$
49. $\sqrt{.01}$	50. $\sqrt{.04}$	51. $\sqrt{\frac{1}{9}}$	52. $\sqrt{\frac{1}{4}}$	53. $\sqrt[3]{\frac{1}{27}}$

1. Square 36.

a.

36
36
$36 = 6 \times 6$
$180 = 6 \times 30$
$180 = 30 \times 6$
$900 = 30 \times 30$
1296

b.

30+6
30+6
180+36
900+180
$900 + 2\ (180) + 36 =$
$30^2 + 2\ (30 \times 6) + 6^2 = 1296$

Learn: The square of any number composed of tens and units is equal to the square of the tens, plus twice the product of the tens and the units, plus the square of the units.

Using the initial letter *t* for tens and *u* for units, we write the algebraic formula for squaring a number thus: $t^2 + 2\,t\,u + u^2$.

2. Square 25, using the algebraic formula.

$25 = 400 + 200 + 25 = 625.$
$t^2 + 2\,t\,u + u^2$

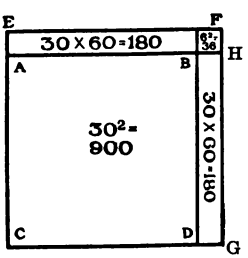
EXERCISES

Using this method, square the following:

3. 44	4. 52	5. 66	6. 75	7. 83
8. 91	9. 14	10. 28	11. 35	12. 42
13. 59	14. 67	15. 54	16. 63	17. 78

18. Square 215.

$215 = 21$ tens $+ 5$ units $= 44{,}100 + 2100 + 25 = 46{,}225.$



The square of a number may be illustrated by a diagram. Let the line A B be 30 units long, and B H 6 units long. Construct on A B a square. What will be its area? Construct on E F a square. How many additions must you make to your first square? What part of the diagram represents the tens' figure squared? The product of the tens' figure by the units' figure? The units' figure squared? Do you see from the diagram why we take *twice* the tens' figure by the units' figure?

19. Draw a diagram and illustrate the square of 18, 26, 32, 48, 65, 84, 73, 98.

1. Find the square root of 144 by factoring.

$144 = 2 \times 2 \times 2 \times 2 \times 3 \times 3$

$2 \times 2 \times 3 = 12.$

If a perfect square is resolved into its prime factors, the square root will consist of each different factor used one half as many times as it appears as a factor.

By factoring find the square roots of the following:

2. 225	**3.** 441	**4.** 324	**5.** 400	**6.** 1225
7. 256	**8.** 484	**9.** 900	**10.** 729	**11.** 1296

Find the square root of 1296.

$$\begin{array}{rrl} & 3\ 6 & \\ & \overline{12\ 96} & \\ & 9\ 00 & = t^2 \\ 2t = 60 & 3\ 96 & = 2tu + \\ 66 & 3\ 96 & u^2 \end{array}$$

Beginning at units' figure, separate 1296 into periods of two figures each. This gives us the number of figures in our root, since in every root there are as many figures as there are periods or fractions of a period in the power. The first period 12 contains the square of the tens' figure. We find the greatest square less than 12 is 9, and its root figure is 3, which we place above the tens' period. Subtract the tens' square. The remainder 396 must contain twice the tens times the units, plus the square of the units. If t is 30, $2t$ will be 60, the *trial divisor*. Dividing by the trial divisor, we find the units' figure 6. Place the 6 above the units' period, and to save one multiplication we also add the units' figure to the trial divisor, giving us the *complete divisor*. Multiply the complete divisor by the last units' figure, and subtract the product from 396. There is no remainder, and the square root is 36.

Find the square root of:

12. 3364	**13.** 6241	**14.** 5929	**15.** 3721
16. 5329	**17.** 9604	**18.** 3025	**19.** 1089
20. 7569	**21.** 2809	**22.** 4356	**23.** 3136
24. 5776	**25.** 8281	**26.** 2704	**27.** 9801
28. 1369	**29.** 5184	**30.** 9025	**31.** 1681
32. 2116	**33.** 6889	**34.** 8836	**35.** 4489
36. 6724	**37.** 1764	**38.** 3364	**39.** 1225
40. 1936	**41.** 2025	**42.** 4096	**43.** 2704

It may make the explanation of square root clearer to use the following diagrams.



FIG 1 FIG 2 FIG 3

Fig. 1 represents a square surface whose area is 1296 square units. Fig. 2 represents the square of the tens' figure, or 900 square units. Fig. 3 represents the part of the original square that remains. This consists of two equal rectangles and one small square, containing 396 square units. It will be seen that these rectangles are 60 units long, or twice the tens' figure. This is our trial divisor. If we divide the area of these rectangles by their length, it will give us the width of the rectangle or the units' figure. If we add this units' figure to the trial divisor, it will give us the length of the two rectangles and the small square. This length is the complete divisor. If we multiply the length by the width, we shall have 396 square units, the area of the remaining part of the original square (Fig. 3). When this is taken away, nothing remains. The square root is 36.

1. *Beginning at the decimal point separate the numbers into periods of two figures each.*

2. *Find the greatest square in the left-hand period, and write its root above the period as the first figure of the root.*

3. *Subtract this square from the left-hand period, and to the remainder annex the next period for a dividend.*

4. *Double the root already found for a trial divisor. Divide by the trial divisor, writing the quotient as the next figure in the root, and adding it to the trial divisor to find the complete divisor.*

5. *Multiply the complete divisor by the last root figure found and subtract the product from the dividend.*

6. *If there is a remainder, annex the next period for a new dividend. Double the root already found, regarded as tens, for a new trial divisor and proceed as before.*

NOTE 1: The process, as given in the rule, should be repeated until all the periods have been used.

NOTE 2: When a number is not a perfect square, annex periods of ciphers until the root has been found to any number of decimal places.

NOTE 3: When a dividend does not contain the trial divisor, place a cipher as the next figure of the root and at the right of the trial divisor. Annex the next period for a new dividend and proceed as before.

NOTE 4: The square root of a common fraction is found by taking the square root of both numerator and denominator. If the denominator is not a perfect square, change the fraction to a decimal and then extract its root.

Find the square root of:

1.	12,544	2.	.027225	3.	$\frac{5}{18}\frac{76}{225}$
4.	77,841	5.	.067081	3.	$\frac{16}{32}$
7.	538,756	8.	285.61	9.	$\frac{225}{256}$
10.	214,369	11.	.082369	12.	$\frac{1}{4}$
13.	390,625	14.	.00143641	15.	$\frac{1}{3}$
16.	948,676	17.	5.000000	18.	$2\frac{1}{2}$
19.	617,796	20.	7.000000	21.	$3\frac{1}{4}$
22.	288,369	23.	11.000000	24.	$5\frac{1}{5}$

25. If the area of a square is 196 square feet, what is the length of one side?

26. How much will it cost at \$1.75 a rod to build a fence round a square field whose area is 10 acres?

Let x = number of rods in one side,
then x^2 = the area or 10 acres = 1600 sq. rd.
$x = \sqrt{1600} = 40$, length of one side.

27. A rectangular lot contains 1875 square yards. If it is 3 times as long as it is wide, what are its dimensions?

Let x = the width of the lot,
and $3x$ = the length of the lot,
then $3x^2 = 1875$ sq. yd. $x^2 = 625$ sq. yd. $x = 25$ yd.

28. A rectangular lot contains 1024 sq. rd. If it is 4 times as long as it is wide, what are the dimensions?

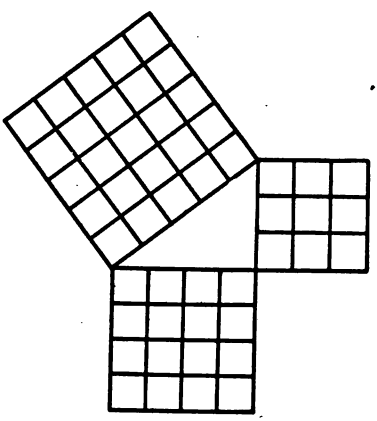
1. How much more will it cost at \$2.50 a rod to fence a rectangular field 312 rd. long and 78 rd. wide, than a square field of the same area?

2. Find the dimensions of a cubical box if the area of its faces is 12,696 square inches.

3. A field contains 63 A. 60 sq. rd. Its length is to its width as 5 to 3. What will it cost to fence it at 72¢ a rod?

4. A triangle containing a right angle is called a right triangle.

5. The **hypotenuse** is the side opposite the right angle. The other two sides are usually called the **base** and **perpendicular.**



6. Draw a right triangle with a base 4 in. and a perpendicular of 3 in. On each side draw a square. Divide each square into square inches.

7. How does the sum of the squares on the base and perpendicular compare with the number on the hypotenuse?

Learn: The square of the hypotenuse is equal to the sum of the squares of the other two sides.

8. We see then that we can formulate a rule for finding any side when the other two sides are given. Letting h stand for the hypotenuse, b for the base, and p for the perpendicular we have the following formulas:

$$h^2 = b^2 + p^2 \qquad h = \sqrt{b^2 + p^2}$$
$$b^2 = h^2 - p^2 \qquad b = \sqrt{h^2 - p^2}$$
$$p^2 = h^2 - b^2 \qquad p = \sqrt{h^2 - b^2}$$

1. State the formulas given on page 225 as rules.

2. The base of a right triangle is 12 ft., and the perpendicular is 16 ft. What is the hypotenuse?

3. The hypotenuse is 25 ft., and the base is 15 ft. Find the perpendicular.

4. The foot of a ladder, 52 ft. long, is placed 20 ft. from the base of a building. How high a window can the top of the ladder reach?

5. Find the length of the rafters of a house 32 ft. wide, if the height of the gable is 12 ft. and the eaves project 1 ft. beyond the walls.

6. A park in the form of a rectangle is 80 rd. long and 72 rd. wide. How many rods long is a walk between the opposite corners?

7. Two vessels sail from the same port. One sails due north 8 miles an hour, and the other due west 6 miles an hour. What is their distance from each other at the end of 5 hours?

8. How many square yards of canvas will make a conical tent 10 ft. in diameter and 15 ft. high?

9. Find the side of a square that shall contain as many square feet as a rectangle measuring 210 ft. by $52\frac{1}{2}$ ft.

10. Find the area of a right triangle whose base is 25 ft. and hypotenuse 60 ft.

11. The slant height of a square pyramid is 15 in., and one side of the base is 24 in. Find its volume.

12. Two poles are 40 ft. apart. One is 50 ft. high and the other 20 ft. How long a line will connect their tops?

13. A ladder 60 ft. long stands 15 ft. from a building, and the upper end rests against the building 4 ft. from the top. How high is the building?

14. Find the side of a square which is equal in area to the sum of the areas of two squares, the sides of which are 12 in. and 16 in. in length.

1. A tree 30 ft. high stands in the center of an island that is 10 ft. in diameter. The center of the island is 10 ft. above the surface of the water. If it is 50 ft. from the top of the tree to the water's edge on the mainland, what width of water separates the island from the mainland?

2. A lot 6 times as long as it is wide contains 18,481½ square feet. What are its dimensions?

3. $\sqrt{11.56} + \sqrt{2.56} = ?$

4. The square of a number, divided by the number, equals 34. What is the number?

5. Cube the square of 9.

6. Find one of the two equal factors of 811,801.

7. $1\frac{1}{4}$ is one of 4 equal factors of what number?

8. Find the square root of 2.5 raised to the fourth power.

9. The perimeter of a rectangular farm is 320 rods. If it is 3 times as long as it is wide, how many acres does it contain?

10. A square field contains 22 acres 80 square rods. Find its perimeter.

11. $\sqrt{2033.1081} = ?$ 12. $\sqrt{3,444,736} = ?$

13. What is the largest square that can be cut from a circular cardboard 32 in. in diameter?

14. When 2255 men were arranged in the form of the largest square possible, 46 men were left over. How many men were in rank and file?

15. A field 4 times as long as it is wide contains $15\frac{5}{8}$ acres. Find its dimensions.

16. A square piece of carpet has an area of 42 sq. ft. 36 sq. in. How long is it?

17. The total area of the sides of a cube is 1422.96 sq. in. What is the length of an edge? What is the volume of the cube?

18. Find to 5 decimal places the square root of 5.

19. Find to 3 decimal places the square root of 4.9.

20. Find to 5 decimal places the square root of 2.

1. If William Andrews of New Haven owes John Blackmer of Chicago a sum of money, he can pay the debt in several ways: *a*, by **postal money order**, or **registered letter**; *b*, by **express money order**; *c*, by **telegraphic money order**; *d*, by **check**; *e*, by **bank draft.**

2. **Exchange** is the payment of money to a person living at a distance, without actually transferring money.

3. **Domestic exchange** is exchange between two places in the same country. **Foreign exchange** is exchange between two places in different countries.

a. **Registered Letter.** Money may be safely sent in a letter if it is registered at the post office. The fee is 10 cents. A receipt is always returned to the sender.

Postal Money Order. A postal money order is an order made by a postmaster in one place, directing the postmaster of another place to pay to the person named therein a specified sum of money. The cost of postal orders is:

Not exceeding $2.50		3¢
Over $2.50 and not exceeding	$5	5¢
Over $5 and not exceeding	$10	8¢
Over $10 and not exceeding	$20	10¢
Over $20 and not exceeding	$30	12¢
Over $30 and not exceeding	$40	15¢
Over $40 and not exceeding	$50	18¢
Over $50 and not exceeding	$60	20¢
Over $60 and not exceeding	$75	25¢
Over $75 and not exceeding	$100	30¢

b. **Express Money Order.** An express money order is similar to a postal money order and the rates are the same.

c. **Telegraphic Money Order.** A telegraphic money order is similar to an express order, but the business is conducted at a telegraphic office instead of an express office. The charge is double the cost of a ten word message, plus 25¢ if the sum does not exceed $25, and plus 1% of the amount of the order for larger sums.

d. **Check.** The check and its use was explained on page 99.

e. **Bank Draft.** A bank draft is a check drawn by one bank on another. The cost of drafts varies.

1. Suppose Mr. William Andrews of New Haven decided to pay John Blackmer of Chicago by bank draft. He would buy a bank draft like the following:

New Haven, Conn., Nov. 10, 1910 No. 4526

The National Tradesmens Bank

Pay to the order of William Andrews $500.00

Five Hundred Dollars

To the National Park Bank,
Chicago.

J. S. Stone.
Cashier

2. Mr. Andrews should then write on the back:
Pay to the order of John Blackmer,
William Andrews.

3. Mr. Andrews should then send the draft to Mr. Blackmer, who should present it to the First National Bank for payment.

4. The person who signs the draft is the **drawer**. The one who is directed to pay is the **drawee**. The one to whom it is to be paid is the **payee**.

5. If the New Haven bank has but little money on deposit in Chicago, it will charge Mr. Andrews a certain per cent or sell the draft at a **premium**. On the other hand, if it has large sums of money there that it wants at home, it will sell Mr. Andrews the draft at a **discount**.

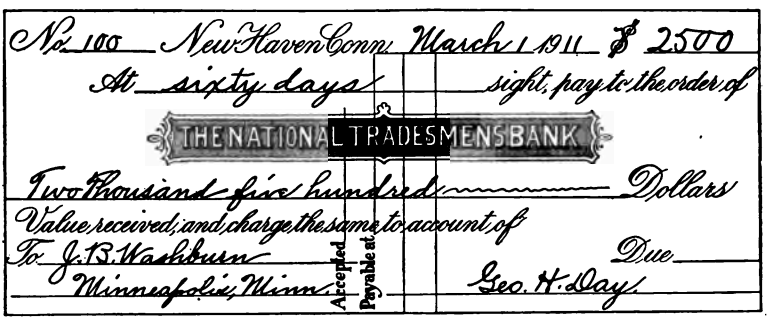
Find the cost of sending the following sums of money (*a*) by post office order, (*b*) by telegraphic order, (*c*) by draft, premium $\frac{1}{2}\%$:

6. \$25. 7. \$50. 8. \$65. 9. \$80. 10. \$100.

11. Find the cost of a draft on New York for \$800, when exchange is $\frac{1}{8}\%$ premium.

1. A **commercial draft** is a draft made by one person or business house on another to secure the payment of a debt.

2. Suppose that George H. Day of New Haven has sold J. B. Washburn, of Minneapolis, an automobile for $2500, who promised to pay the bill in 60 days. When Mr. Day sends the bill he sends also through a bank a draft like the following:



3. If Mr. Washburn accepts it, he must write the word "Accepted" with the date and his signature across the face, and return the draft to Mr. Day. Mr. Day may now discount this at a bank as he would a promissory note.

4. The draft above is a **time draft.** Instead of the words "sixty days after," the draft might read "at sight." This draft would be called a **sight draft.** When presented to Mr. Washburn, he will not need to write accepted, as it is due at once.

5. If Mr. Washburn refused to pay the draft it would be returned to the New Haven bank and Mr. Day would be notified.

6. In discounting drafts, banks in some places allow three days of grace to the term of discount for time in which to collect the money. In this book they will not be considered.

7. Banks may charge for this exchange. The rate of exchange varies, as was explained on page 229.

1. Find the cost of a draft of \$700, payable at sight, premium $\frac{1}{2}\%$.

2. A draft, dated June 8, at 30 days sight for \$650, was accepted June 11, and discounted on June 16, at 6%. What were the proceeds?

3. The Winchester Repeating Arms Co. sold goods listed at \$1750, with discounts of 10% and 5%, and drew on the buyer at 60 days sight. The buyer accepted the draft, and it was discounted 15 days later. Write the draft and find the proceeds.

4. A grain dealer in Minneapolis sold some grain to a Boston firm and drew on them at sight for \$8250. The draft was accepted and a Minneapolis bank bought it at a discount of 40¢ per \$1000. How much did the grain dealer receive?

5. What is the cost of a draft on New York for \$850, when exchange is at $\frac{1}{4}\%$ premium? Find the cost of the same draft, when exchange is at $\frac{1}{4}\%$ discount.

6. What is the cost of a draft for \$2500 on San Francisco, when exchange is at $\frac{1}{2}\%$ discount?

7. A wholesale house sold goods, listed at \$3500, with discounts of 8% and 4%, and drew on the buyer at 90 days sight. The buyer accepted the draft, and it was discounted 15 days later at 6%. If the bank also charged 20¢ per \$1000 for exchange, give the net proceeds.

8. At 75¢ discount per \$1000, how much will a sight draft on Chicago for \$1200 cost?

9. Which is cheaper for you, if you owe a merchant \$80 for some goods, to send an express order, or a draft for which you pay $\frac{1}{2}\%$ premium? How much cheaper is it?

10. If you owe in a neighboring city a bill of \$20, is it cheaper for you to send it by registered letter or money order? How much cheaper?

11. Benson & Co. draw on F. C. Keyes for \$450. If the bank charges 1% for collection, what are the proceeds?

1. A **foreign bill of exchange** is similar to a bank draft and is payable in the money of the country on which it is drawn.

2. To secure safety and speed in the transmission of foreign bills of exchange, they are drawn in sets of two or three of the same tenor and date. The separate bills are sent by different steamers, and when one is paid, the others become void.

3. The following is the par value of exchange as determined by the U. S. Mint:

1 pound = $4.8665	**1 mark = $0.238**
1 franc = $0.193	**1 lira = $0.193**

4. Newspapers usually give exchange rates for cable transfers, demand bills, and sixty-day bills. These quotations vary as the balance of indebtedness between the countries varies.

5. Suppose you read in the paper, sterling, 4.88; francs, 5.155; marks, $95\frac{1}{2}$. This means that you can buy 1 pound sterling for $4.88; 5.155 francs for $1; 4 marks for $95\frac{1}{2}$ cents.

Using the market prices quoted above, find the cost of:

6. £475	7. 6186 francs	8. 1600 marks
9. £600	10. 5155 francs	11. 2400 marks

12. Find the cost of a bill of exchange on Paris for 800 francs at $5.16\frac{1}{4}$ for $1.

13. Find the cost of a bill of exchange on Berlin for 2680 marks at 96.

14. A debt of $8398.48 is owed by a London firm to a New York firm. For how many pounds must a draft be made?

15. How large a draft on London can be bought for $10,079.25 when exchange is 4.91?

16. How large a demand draft on Paris will $375 buy when exchange is 5.20?

1. The **metric system** is a decimal system of weights and measures, which originated in France.

2. The use of the metric system is permitted by law in the United States. Since it is a decimal system, it is much simpler than our system, and is especially adapted for scientific work.

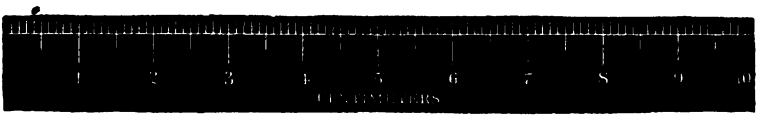
3. The **meter** is the primary unit of length, and to it all the other units of the system are related.

4. The word meter means *measure*. It was intended that the meter should be one ten-millionth of the distance from the equator to the poles, but is found to be slightly less than that. It is 39.37 in. in length.

5. To indicate lengths less than a meter, Latin prefixes are used. To indicate lengths longer than the meter, Greek prefixes are used.

Latin prefixes	Greek prefixes
milli means $\frac{1}{1000}$	**deka** means 10
centi means $\frac{1}{100}$	**hekto** means 100
deci means $\frac{1}{10}$	**kilo** means 1000
	myria means 10,000

One Decimeter



MEASURES OF LENGTH

10 millimeters (mm)	= 1 centimeter (cm)
10 centimeters	= 1 decimeter (dm)
10 decimeters	= 1 meter (m)
10 meters	= 1 dekameter (Dm)
10 dekameters	= 1 hektometer (Hm)
10 hektometers	= 1 kilometer (Km)
10 kilometers	= 1 myriameter (Mm)

1. A millimeter is what part of a centimeter?

2. A centimeter is what part of a decimeter?

3. A millimeter is what part of a decimeter?

4. Notice that any measure may be expressed in terms of another measure by simply moving the decimal point to the right or left.

Express in meters and decimals of a meter:

5. 8 Km	6. 116 dm	7. 410 cm
8. 4 Hm	9. 716 dm	10. 1214 mm
11. 2 Mm	12. 221 dm	13. 2480 mm

14. Measure off 10 meters on a string, with knots to indicate the meters. Find the length and breadth of the school yard in dekameters.

15. Change 59.055 in. to meters. 16. Change 50-ft. to meters.

17. Change 15 m. to feet. 18. Change 45 dm. to feet.

19. Change 475 cm. to feet. 20. Change 1450 mm. to feet.

SURFACE MEASURES

21. Draw a square decimeter. Divide it into square centimeters. How many are there?

22. In the same way find out how many square millimeters there are in a decimeter.

23. The **square meter** is the primary unit of surface. It is equal to 1.196 square yards.

TABLE OF SURFACE MEASURES

100 square millimeters = 1 square centimeter.
100 square centimeters = 1 square decimeter.
100 square decimeters = 1 square meter.
100 square meters = 1 square dekameter.
100 square dekameters = 1 square hektometer.
100 square hektometers = 1 square kilometer.

1. In measuring land the square dekameter is called an **are**, and the square hektometer is called a **hektare** (Ha). It is equal to 2.47 acres, or nearly $2\frac{1}{2}$ acres.

2. If you multiply square meters by 10,000, to what denomination have you changed them?

3. In a 2-meter square there are how many square decimeters?

4. In a square decimeter there are how many square millimeters?

5. Find the area of a rectangular surface that is 1.5 m. long and 8 decimeters wide.

6. Write 28 sq. dm. as a decimal of a square meter.

7. How much larger is a rectangle measuring 4 m. by 3 m. than one measuring 4 yd. by 3 yd.?

8. How many square centimeters are there in a rectangle 5 dm. by 6 dm.?

MEASURES OF VOLUMES

9. If each edge of a cube is a decimeter long, how many cubic decimeters does it contain?

10. Each edge of this same cube is how many centimeters long? Then the cube contains how many cubic centimeters?

TABLE

1000 cubic millimeters = 1 cubic centimeter.
1000 cubic centimeters = 1 cubic decimeter.
1000 cubic decimeters = 1 cubic meter.

11. Reduce

a. 13.46 cu. m. to cubic decimeters.
b. 12 cu. m. to cubic centimeters.
c. 4120 cu. cm. to cubic meters.
d. 48,675 cu. dm. 46 cu. cm. to cubic meters.
e. 48 cu. m. to cubic millimeters.
f. 17,280 cu. mm. to cubic centimeters.

1. In measuring wood the cubic meter is called a **stere** (**st**).

2. How many cubic meters are there in a box 1.4 m. long, 1.1 m. wide, and .6 m. deep?

3. How many cubic decimeters are there in a wall 45 m. long, 26 dm. high, and 246 mm. thick?

4. How many bricks, 20 cm. by 10 cm. by 5 cm., will it take to build a solid wall 60 m. long, 2.5 m. high, and 65 cm. thick?

5. How many steres of wood are there in a cubical pile, each edge of which measures 8 m.?

6. Find the total surface and volume of a cube, if the edge measures 25 dm.

7. How many square meters of sheet tin are required to cover the convex surface of a circular tower 40 m. high, if the diameter of the base is 8 m.? Find the number of cubic meters of water that the cylinder will hold.

MEASURES OF CAPACITY

8. The **liter** is the unit of capacity for both liquid and dry measures. It contains 1 *cubic decimeter.*

9. The liter is equal to .908 qt. dry measure or 1.0567 qt. liquid measure.

10. Substitute the word liter for meter in the table for length on page 233, and write the table of measures of capacity.

11. How many cubic centimeters are there in a liter?

12. Write 65 liters as dekaliters; as deciliters; as centiliters; as hektoliters.

13. How many hektoliters are there in a bin 4 m. long, 3 m. wide, and 2 m. deep?

14. How many liters are there in a tank 5.6 m. long, 3.25 m. wide, and 1.4 m. deep?

15. How many liters are there in a tank 4.5 m. long, 2.75 m. wide, and 1.25 m. deep?

1. The **gram** is the metric unit of weight. It is the weight of a *cubic centimeter* of pure water.

2. The prefixes used with the terms meter and liter are used with the gram. Give the name of the tenth of a gram; of the hundredth of a gram; of 100 grams.

3. Write the table of metric weights.

4. A kilogram is equivalent to 2.2046 lb. (about $2\frac{1}{5}$ lb.), and 1000 kilograms equal 1 metric ton (about 2205 lb.).

5. Change 450,600 cg. to kilograms.

6. Add 44 dg., 4.638 Dg., and 2.45189 Hg.

7. Divide 148,680 g. by 6.3, and express the result in kilograms.

8. What will 474.6 Hg. of beef cost at 28¢ a kilogram?

9. At $6.50 a ton, what will the coal cost to keep a fire a week if 30 kilos (kilograms) are burned each day?

10. Find the weight of water that may be contained in a cistern 4 m. deep, 1.5 m. long, and 1.2 m. wide.

11. The United States imported from Greece in one year 25,000 metric tons of currants. Find the value at 15¢ a pound.

12. If 3 metric tons of cotton are produced on a hektare, find the value of 6.5 Ha. of cotton at 15¢ a kilogram.

13. A rectangular tank 4.8 dm. long, 25 cm. wide, contains 56 liters of water. What is the depth of the water? Find the weight of the water in kilograms and in pounds.

14. How many metric tons of water will it take to fill a cylindrical tank, 15 m. in diameter, to a depth of 6 m.?

15. What is the weight of 15 cu. cm. of lead, if lead is 11.35 times as heavy as water?

16. Find the cost of 2.5 metric quintals of chestnuts imported at 8 francs per sack, each sack containing 50 kilos. A metric quintal is equal to 100 kilograms.

17. Multiply 2.48 dg. by 24 and express the result in dekagrams.

1. What is the duty, at 22¢ a gallon, on 400 quart-bottles of olive oil?

2. Find the convex surface of a cylinder whose altitude is 8 ft., and the circumference of the base 8 ft.

3. What single discount is equivalent to a discount of 25% and 10%?

4. What tax must a man pay on his property, valued at \$8000, if the rate of taxation is $1\frac{1}{2}$%?

5. An agent sold \$14,000 worth of goods, and received \$280 as his commission. What was the rate of commission?

6. What is the rate of taxation if \$74,240 is raised on real estate valued at \$7,424,000?

7. A father's age is 4 times that of his son, and their ages together amount to 80 years. How old is the son?

8. A man who owned $\frac{3}{5}$ of a mill sold $\frac{1}{3}$ of his share for \$20,000. Find the cost of the mill.

9. A high school building is insured for \$36,000, which is $\frac{3}{4}$ of its value. Find its value. If the rate is 1%, find the premium.

10. What per cent of \$600 is \$12?

11. What is 50% of 12 rd. 2 yd. 2 ft.?

12. What is a prime number? Name one.

13. If 3 men can do a piece of work in $12\frac{3}{4}$ days, how long will it take 9 men to do it?

14. When $\frac{1}{2}$ of a number is added to the number itself, the sum is 24. What is the number?

15. What per cent of 12 bu. is 6 pk.?

16. Two boys have 28 apples. One has $\frac{1}{3}$ more than the other. How many has each?

17. At 40¢ a gallon, how much will 6 gal. 2 qt. cost?

18. What per cent of \$24 is \$18?

19. 210 is $16\frac{2}{3}$% more than what number?

20. What number must be added to $18\frac{1}{6}$ to make $24\frac{5}{6}$?

1. The difference between two numbers is 11, and their sum is 27. What are the numbers?

2. Change 23.45 Dg. to decigrams.

3. Find the volume of a square pyramid, if the height is 4.5 cm. and each side of the base .8 dm.

4. A hollow cannon ball measures 5 dm. in diameter. If the diameter of the hollow part is 4 dm., find the volume of iron in the ball.

5. Find the area of a trapezoid if the parallel sides are 4 m. and 300 cm., and the altitude 1.8 m.

6. The sides of a rectangle are 28 m. and 63 m. Find the side of a square, equivalent to the rectangle.

7. The perimeter of a rectangle is 120 m. If the length is twice the breadth, find the length, breadth, and area.

8. The perimeter of a rectangle is 56 m., and the length is to the breadth as 4 to 3. Find the length, breadth, and area.

9. The area of a rectangle is 288 sq. m., and the breadth is 8 m. Find the length and the perimeter.

10. In grading a lot 162 ft. long and 40 ft. wide, it was found necessary to raise it 15 in. How many cubic yards of earth were needed?

11. It cost \$260 to inclose a rectangular field 90 rd. long and 40 rd. wide. How much will it cost to inclose a square field of the same area with the same kind of a fence?

12. The perimeter of a square and the circumference of a circle are each 16 rd. Which has the greater area? How much greater is it?

13. A 3 mo. note for \$1632, dated July 1, was discounted Sept. 5. Find the proceeds.

14. At compound interest, what is the amount of \$1600 at 6% for 5 yr. 8 mo.?

15. A man invested \$45,250 in $3\frac{1}{2}\%$ stock at 113, brokerage $\frac{1}{8}\%$. What income did he receive?

1. One side of a square field of $22\frac{1}{2}$ A. abuts on a road. This side is divided into building lots 110 ft. deep, having a frontage along the road of 90 ft. each. If the building lots are sold at \$250 each, and the rest of the field at \$75 an acre, find the total amount received.

2. A man obtained an income of \$60 from an investment of \$1560 in U. S. 5's. What was the market price of the bonds?

3. A man bought some railroad stock at 60% premium, paying \$19,200. How many shares did he buy?

4. Find the square root of 3622.8361.

5. A room is 32 ft. long, 20 ft. wide, and 14 ft. high. There are 3 doors, each 8 ft. by 4 ft., and 4 windows, each 7 ft. by $3\frac{1}{2}$ ft. Find the cost of plastering the room at $12\frac{1}{2}$¢ a square yard.

6. The longitude of A. is 110° 15′ 40″ W., and the longitude of B. is 25° 5′ 20″ E. When it is 2.15 A.M. at A., what time is it at B.?

7. When it is 10.15 A. M. at A., it is 1.30 P.M. at B. If the longitude of B. is 40° 10′ 15″ W., what is the longitude of A.?

8. What is the interest on \$465.82 from May 15, 1912, to Jan. 6, 1914, at $4\frac{1}{2}$%?

9. A merchant sold goods to the amount of \$31,378 this year. If this was $8\frac{1}{5}$% more than he sold last year, what was the amount of the sales last year?

10. A block of granite is 38 ft. long and $9\frac{1}{2}$ ft. square at the ends. How many cubic feet must be cut away to leave a perfectly cylindrical pillar?

11. Elliot and Davey engaged in trade with \$6000 each. At the end of 3 mo. they admitted Martin with a capital of \$10,000. If the total gain for the year was \$3276, find each partner's gain.

12. Find the square root of 1866.24.

13. Find the square root of 25.8064.

1. How many bushels of shelled corn are there in a wagon bed 3 ft. wide, 9 ft. long, and 2 ft. 3 in. deep?

2. A farmer owns a farm of 80 acres. The pasture occupies a field 20 rd. by 25 rd., and the buildings a plot 20 rd. by 30 rd. A road occupies a piece 2 rd. wide and 160 rd. long. If the rest of the farm is planted to potatoes, how many acres of potatoes will there be?

3. At $75 an acre, find the cost of a piece of land 12 rd. wide and 80 rd. long.

4. How many bushels of oats will fill a bin 10 ft. wide, 12 ft. long, and 12 ft. deep?

5. A field is 120 rd. by 80 rd. If a man charges $3 a day for himself and his team, and can plow 3 acres a day, find the cost of plowing the field.

6. If a man and team can plant 12 acres a day, what will it cost to plant this field?

7. Eight pounds of seed corn will plant an acre. How many bushels of corn will plant this field? A bushel of corn weighs 56 lb.

8. If this field of corn yields 4000 bushels, what is the average yield an acre?

9. A flock of 200 hens averages 120 eggs each a year. If the average price for a year is 20¢ a dozen, find the value of the eggs from the flock.

10. If it takes 24 bu. of corn at 45¢, 10 bu. of oats at 28¢, and $12 worth of other feed for the flock for the year, what is the profit from the flock for the year?

11. Find the value of 4 loads of hay sold at $13.50 a ton. The loads weighed 3240 lb., 3360 lb., 3410 lb., 3350 lb., and the wagons alone weighed 1020 lb., 980 lb., 1180 lb., 1210 lb.

12. A 20-acre field measures 80 rd. in length. How many rods of tile will it take for a drain, running diagonally across the field?

13. A man's farm was 120 rd. wide. He sold 12 A. off from one end. How much shorter was his farm than at first?

1. What is the square root of 529? Of 1296?

2. What is square root? What is a power?

3. When it is 12 M. at Boston, is it earlier or later by solar time at Albany?

4. If 10 men can do a piece of work in 30 days, how many men can do the same work in 15 days? In 60 days?

5. What is proportion?

6. Test the following proportion in two ways to prove whether or not it is correct: $6 : 10 = 4 : 3$.

7. What will 20 shares of Interurban stock cost at 80?

8. What is the difference between the par value and the market value of a share of stock?

9. What is exchange?

10. State several ways in which exchange of money may be carried on.

11. State the three steps necessary to take in finding the bank discount on any non-interest bearing note. If the note bears interest, what other step is necessary?

12. What is interest?

13. Find the interest on \$400 for 3 yr. at 4%.

14. What is insurance? What is the premium?

15. What annual premium must be paid on a policy of \$4000 at $\frac{4}{5}$%?

16. Given the amount of taxable property and the tax to be raised, what can you find? How do you find it?

17. What is a poll tax?

18. What is meant by the net amount of a bill?

19. A purchaser was allowed a discount of 30% on a typewriter listed at \$100. What price was paid?

20. What is commission? On what is an agent allowed to take a commission?

21. An agent sold \$10,000 worth of goods on a commission of 2%. Find the commission.

1. Find the square root of 14 to three decimal places.

2. Find the square root of 987,656,329.

3. A traveler, on arriving at Cleveland, found his watch 1 hr. too slow. From what direction and from what time belt had he come?

4. If $7\frac{1}{6}$ acres of land yield 344 bu. of wheat, how much will $18\frac{3}{4}$ acres yield?

5. A manufacturing company, whose capital stock was \$250,000, declared a semi-annual dividend of $3\frac{1}{2}\%$, and placed \$5706.50 in the reserve fund. What were the net earnings?

6. What is the cost of a draft for \$1080 on New Orleans at $\frac{1}{4}\%$ premium?

7. The face of a note is \$825. If it was discounted for 45 days at 6%, find the proceeds.

8. A 3 mo. note for \$1210 was dated Sept. 1, 1911, and discounted Sept. 18, 1911, at 6%. If the note bore interest at 6%, find the proceeds.

9. Find the interest on \$213.50 at 5% for 5 yr. 6 mo. 21 da.

10. Find the interest on \$2050 at $4\frac{1}{2}\%$ from Aug. 28, 1911, to Feb. 19, 1815.

11. A ship worth \$156,000 was insured for $\frac{3}{4}$ of its value at $2\frac{1}{2}\%$. The cargo, worth \$25,920, was insured for $\frac{2}{3}$ of its value at $3\frac{1}{2}\%$. Find the total premium.

12. A tax of \$30,754.84 is to be raised in a town, the taxable property of which is valued at \$3,344,772. If there are 1056 polls, each taxed at \$2, find the total tax a man must pay who owns property valued at \$12,150.

13. Find the net amount of a bill of \$450, the discounts being 20% and 4%.

14. After discounts of 25% and 20% had been allowed, a piano was sold for \$360. What was the list price?

15. What is an agent's commission at $1\frac{3}{8}\%$ for collecting \$3793.56? What amount should he return to his employer?

1. One half of a man's age added to twice his age equals 45 years. Find the man's age.

2. The perimeter of a rectangular field is $20x$ rods. If it is $4x$ rods wide, how long is it?

3. If $3x - 2$ represents an even number, what will represent the next smaller even number?

4. What is the interest on a dollars for b years at 4% a year?

5. If a man lives x years, he will be a years old. How old is he now?

6. Add $6abc$ and $-4abc$.

7. The sum of two numbers is 40, and their difference is 10. Find the numbers.

8. A has 3 times as many cows as B. If they together have 28, how many has each?

9. What is a sphere? Give the rule for finding the volume of a sphere, when you know its diameter.

10. Find the convex surface of a cone whose circumference is 24 ft., and whose slant height is 20 ft.

11. Find the convex surface of a cylinder whose circumference is $6\frac{1}{2}$ ft., and whose altitude is 12 ft.

12. Find the area of a trapezoid whose parallel sides are 25 ft. and 15 ft., and whose altitude is 12 ft.

13. The area of a square field is 10 acres. How long is each side?

14. The difference in longitude between two places is 60° 30′. What is the difference in time?

15. The difference in time between two places is 3 hr. Find the difference in longitude.

16. If \$40 is paid as commission for selling \$2000 worth of property, what is the rate of commission?

17. How many pounds are there in 25% of a ton?

18. What per cent of a number is $\frac{3}{20}$ of it?

19. Find the cost of 27 lb. of butter at $33\frac{1}{3}$¢ a pound.

1. A rectangle has the same dimensions as the diameter and circumference of a sphere whose radius is 12 inches. Compare the area of the rectangle with the surface of the sphere.

2. A field of corn is 80 rd. long and 60 rd. wide. The rows are 3 ft. 8 in. apart. How many rows are there lengthwise of the field?

3. If the hills are 3 ft. 8 in. apart, how many hills of corn are there in the field?

4. If there is an average of 5 ears of corn to each hill and 100 ears make a bushel, how many bushels of corn will this field produce?

5. The corn from the field is put into a bin 54 ft. long and 9 ft. wide. How deep must the bin be to hold the corn?

6. At 45¢ a bushel find the value of the corn.

7. Change $.44\frac{4}{9}$ and $.42\frac{6}{7}$ to common fractions.

8. The minuend is 300.0019, and the subtrahend is 115.4079. What is the difference?

9. Find the cost of plastering the walls and ceiling of a room 27 ft. long, 21 ft. wide, and 12 ft. high, at 27¢ a square yard, making no allowance for openings.

10. What is the capacity in gallons of a tank 9 ft. long, 7 ft. 4 in. wide, and 5 ft. 6 in. deep?

11. Divide four million by two thousand, and multiply the quotient by sixteen thousandths.

12. Find the value of a pile of wood 64 ft. long, 4 ft. wide, and 5 ft. high, at $5.75 a cord.

13. In a certain town the valuation of the property amounts to $1,720,000. The town raised $28,780 by taxation. There are 840 persons, each assessed a poll tax of $1.50. Find the rate of taxation on $1000.

14. The face of a note is $1390; the date, Dec. 15; the time, 90 da.; the day of discount, Dec. 15; the rate 6%. Find the proceeds.

NOTE : The remaining pages of this book may be omitted without loss. The topics are inserted here for convenience, if required to be taught.

Time, Rate, and Interest Given, to Find the Principal.

1. What principal, at 6%, will yield $225 interest in 2 yr. 6 mo.?

$.15 = int. of $1 for 2 yr. 6 mo.
$225 ÷ .15 = $1500.

Divide the given interest by the interest of $1 for the given time and rate.

Find the principal that will gain:

2. $80 in 3 yr. 4 mo. at 6%.
3. $400 in 7 mo. 15 da. at 5%.
4. $15 in 2 yr. at 6%.
5. $20 in 4 yr. at 5%.
6. $65 in 5 yr. at 5%.
7. $792 in 2 yr. at $4\frac{1}{2}$%.

Principal, Interest, and Time Given, to find the Rate.

8. At what rate will $900 gain $231 in 3 yr. 8 mo.?

$33 = int. on $900 for 3 yr. 8 mo. at 1%.
$231 ÷ $33 = 7 ∴ 7%.

Divide the given interest by the interest on the principal for the given time at 1%.

Find the rate when you know the following:

	Principal	Interest	Time
9.	$426	$63.90	2 yr. 6 mo.
10.	$120	$24.92	3 yr. 5 mo. 16 da.
11.	$600	$81	3 yr.
12.	$340	$59.50	3 yr. 6 mo.
13.	$7250	$435	1 yr. 6 mo.
14.	$240	$45.90	4 yr. 3 mo.
15.	$800	$192	3 yr.

16. Given the principal, rate, and time, how do you find the interest?

17. Given the time, rate, and interest, how do you find the principal?

18. Given the principal, interest, and time, how do you find the rate?

Principal, Interest, and Rate Given, to Find the Time.

1. In what time will \$940 gain \$432.40 at 6%?

\$56.40 = int. of \$940 for 1 yr. at 6%.
\$432.40 ÷ \$56.40 = $7\frac{2}{3}$.
∴ 7 yr. 8 mo.

Divide the given interest by the interest on the principal for 1 yr. at the given rate.

Find the time when you know the following:

	Principal	Interest	Rate		Principal	Interest	Rate
2.	\$500	\$5	4%	3.	\$254	\$44.45	5%
4.	\$900	\$13.50	6%	5.	\$75	\$15.80	4%
6.	\$725	\$10.15	$3\frac{1}{2}$%	7.	\$850.50	\$136.08	5%

8. How long will it take \$200 to double itself at 4%?

9. How long will it take \$1115.36 to amount to \$1204.838 at $4\frac{1}{2}$%? Hint: First find the interest.

PARTIAL PAYMENTS

10. **Partial Payments** are payments of a part of a note, made at different times.

11. A record of each partial payment, with date of payment, is made on the back of the note. Such records are called **indorsements.**

MERCHANT'S RULE

12. This rule is usually used by bankers and business men on notes that do not run longer than a year.

13. A note of \$850 was dated Jan. 2, 1911. The indorsements were March 18, \$200; May 2, \$150; Aug. 18, \$300. What was due Dec. 2, 1911?

Amt. of \$850 from Jan. 2 to Dec. 2		\$896.75
Amt. of \$200 from March 18 to Dec. 2	\$208.47	
Amt. of \$150 from May 2 to Dec. 2	155.25	
Amt. of \$300 from Aug. 18 to Dec. 2	305.20	668.92
		\$227.83

Study this solution till you see clearly each step taken in solving a problem by the Merchant's Rule.

Merchant's Rule: *a. Find the amount of the principal at the date of settlement.*

b. Find the amount of each payment from the time it was made to the date of settlement.

c. From the amount of the principal subtract the sum of the amounts of the payments.

1. A note of \$1250.60, dated July 6, 1911, with interest at 6%, was indorsed as follows: Sept. 21, 1911, \$260; Nov. 22, 1911, \$325; March 6, 1912, \$120; May 17, 1912, \$250. What was due at settlement July 6, 1912?

2. A note of \$2000, dated Jan. 20, 1912, had the following indorsements: May 20, \$100; July 20, \$175; Dec. 20, \$250. Find the balance due Jan. 20, 1913.

UNITED STATES RULE

Find the amount of the principal to the time when the payment or sum of the payments equals or exceeds the interest, and from this amount deduct the payment or sum of the payments.

Consider the remainder as a new principal and proceed as before.

3. A note of \$690 was dated Feb. 3, 1912. The indorsements were Dec. 3, 1912, \$40; April 3, 1913, \$90. What was due Feb. 27, 1914?

Face of the note	\$690.00
Int. from Feb. 3, '12, to Dec. 3, '12 (10 mo.)	34.50
Amount due Dec. 3, '12	\$724.50
First payment	40.00
Balance due Dec. 3, '12, or new principal	\$684.50
Interest from Dec. 3, '12, to Apr. 3, '13 (4 mo.)	13.69
Amount due April 3, '13	\$698.19
Second payment	90.00
Balance due April 3, '13, or new principal	\$608.19
Interest from Apr. 3, '13 to Feb. 27, '14 (8 mo. 24 da.)	26.76
Amount due Feb. 27, '14	\$634.95

Solve the following by U. S. rule. In case the note is settled within a year, solve it also by Merchant's rule:

1. Face, $460. Date, May 9, 1913. Settled, Feb. 24, 1914. Payments: July 1, 1913, $120; Sept. 16, 1913, $150; Jan. 2, 1914, $100.

2. Face, $165. Date, April 15, 1912. Indorsements: May 24, 1913, $24.18; July 18, 1914, $5.25; Sept. 6, 1914, $45; Jan. 24, 1915, $40. What was due April 15, 1916?

3. Face, $1600. Date, Sept. 16, 1911. Indorsements: June 8, 1912, $400; Oct. 15, 1913, $500; Jan. 15, 1914, $300. With interest at 5%, what is due May 10, 1914?

4. A note for $3000 with interest at $4\frac{1}{2}\%$, dated Jan. 1, '11, was indorsed as follows: Mar. 7, '11, $175; May 9, '12, $300; Aug. 17, '13, $400; Aug. 17, '14, $400; Nov. 30, '14, $800. What was due at date of settlement, Dec. 5, 1914?

5. A note of $2000, dated Jan. 20, 1912, had the following indorsements: May 20, 1912, $100; July 20, 1912, $175; Dec. 20, 1912, $250. Find the balance due Jan. 20, 1913.

6. A note of $720, dated Aug. 14, 1911, has the following indorsements: Dec. 26, 1912, $200; Sept. 14, 1913, $175; Dec. 31, 1913, $400. If the note was settled Dec. 31, 1914, find the sum paid at settlement.

7. A note for $3000, dated Jan. 1, 1912, had indorsements as follows: March 1, 1912, $300; Oct. 1, 1912, $300; Nov. 1, 1912, $1200. Find the balance due Jan. 1, 1913.

8. A note of $1200, dated Aug. 15, 1911, was indorsed as follows: Dec. 15, 1911, $20; Sept. 15, 1912, $150. Find the amount due Aug. 15, 1913.

9. A note of $800 with interest at 6% was given March 4, 1911. The following payments were made: April 12, 1911, $75; July 9, 1911, $150; Sept. 5, 1911, $90; Dec. 8, 1911, $200. What was due Jan. 1, 1912?

1. The Government and some banks in computing interest take the exact number of days between dates and reckon 365 days to a year.

2. Interest computed in this way is called **exact** or **accurate interest.**

3. Find the interest of \$300 from July 7, 1911, to Oct. 25, 1911, at 6%.

\$300	In July	24 days
.06	In Aug.	31 days
\$18.00 int. for 1 yr.	In Sept.	30 days
	In Oct.	25 days
$\frac{22}{73}$ of \$18 = \$5.43	110 days = $\frac{110}{365}$ = $\frac{22}{73}$ of a year.	

4. Find the interest of \$4000 from March 5, 1911, to July 10, 1912, at 7%.

From March 5, 1911, to March 5, 1912 = 1 year
From March 5, 1912, to July 10, 1912 = 127 days = $\frac{127}{365}$ of a year.

\$4000	$\frac{127}{365}$ of \$280 = \$97.43
.07	
\$280.00 = int. for 1 yr.	
97.43 = int. for 127 days.	
\$377.43	

Find the exact interest of:

5. \$650 from Oct. 17, 1911, to April 11, 1912, at 6%.
6. \$1100 from Jan. 7, 1911, to Sept. 3, 1911, at 5%.
7. \$1500 from Aug. 9, 1911, to Feb. 13, 1912, at $4\frac{1}{2}$%.
8. \$275 from Mar. 10, 1911, to June 17, 1912, at 5%.
9. \$180 from Feb. 6, 1912, to Aug. 19, 1913, at 6%.
10. \$25,000 from May 1, 1911, to Dec. 3, 1911, at $4\frac{1}{2}$%.
11. \$50,000 from July 5, 1911, to May 7, 1912, at 4%.

1. In lesson 221 we learned that $25^2 = (20 + 5)^2 = (20)^2 + 2\ (20 \times 5) + 5^2$. To cube a number we must multiply the square of the number by the number itself.

$$\begin{array}{r} 20^2 + 2\ (20 \times 5) + 5^2 \\ \underline{\qquad\qquad\qquad\qquad\qquad\quad 20 + 5} \\ (20^2 \times 5) + 2\ (20 \times 5^2) + 5^3 \\ 20^3 + 2\ (20^2 \times 5) + \quad (20 \times 5^2) \\ \underline{\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad} \\ 20^3 + 3\ (20^2 \times 5) + 3\ (20 \times 5^2) + 5^3 \\ t^3 + 3t^2u \qquad\quad + 3tu^2 \qquad\quad + u^3 \end{array}$$

Every perfect cube consists of four parts:

a. The tens' figure cubed.

b. Three times the tens' figure squared times the units' figure.

c. Three times the tens' figure times the units' figure squared.

d. The units' figure cubed.

2. Cube 36, using the formula.

$36^3 = 30^3 + 3\ (30^2 \times 6) + 3\ (30 \times 6^2) + 6^3 =$
$27{,}000 + 16{,}200 \quad + 3240 \quad + 216 = 46{,}656$

3. Using the formula, write the cube of 74, 48, 63.

4. Extract the cube root of 46,656.

$$\begin{array}{rl} & 3\ 6 \\ & \overline{46{,}656} \\ & 27 \quad = t^3 \\ 3t^2 = 27) & \overline{196} \\ & 162 = 3t^2u \\ & \overline{345} \\ & 324 = 3tu^2 \\ & \overline{216} \\ & 216 = u^3 \\ & \overline{\quad} \end{array}$$

We first point off the number into periods of three figures each. The first period, 46, contains the cube of the tens' figure. The greatest cube in 46 is 27, which is the cube of 3. Place the 3 above the tens' period. Cube the tens' figure and subtract it from the tens' period. To the remaider annex the first figure of the next period. The next part of the formula is $3t^2u$, of which we can find the value of $3t^2$. This is our *trial divisor*, and is equal to 3 times the square of the tens' figure. Dividing 196 by 27, we find the units' figure of our root is 6. Place it over units' period. Find the value of $3t^2u$ an subtract it from 196. To the remainder 34, annex the next figure of the power. Fin ' the value of $3t^2u$ and subtract it from 345. To the remainder 21, annex the last figure of the power. Find the value of u^3 and subtract it from 216. There is no remainder. The cube root of 46,656 is 36.

1. Find the cube root of 15,625.

Fig. 1.

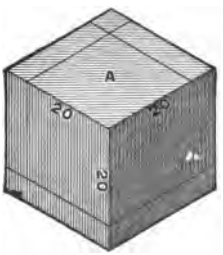
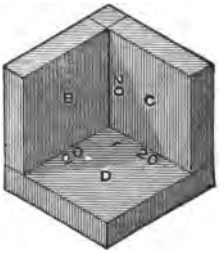


Fig. 2.



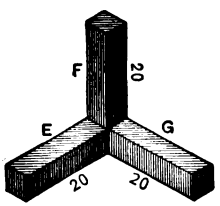


Fig. 3.

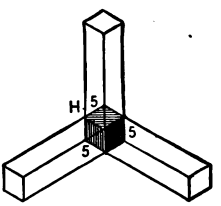


Fig. 4.

The entire cube in Fig. 1 represents 15,625.

The part marked A represents the largest tens' figure cubed. Fig. 2 represents the part left after the tens' cube has been removed. B, C, and D are three solids, each as long and wide as the tens' cube. $3 \times (20)^2 \times 5 =$ contents of B, C, and D. Fig. 3 represents what is left after B, C, and D have been taken away from Fig. 2. How long, wide, and thick are E, F, and G? What, then, are their cubic contents? Fig. 4 represents the part of Fig. 3 that is left after E, F, and G are removed. This is a little cube. What are its dimensions? What is its cubic contents? What part of the formula represents each of the eight parts of the cut-up cube?

2. Find the cube root of:

a. 54,872	*b.* 804,357	*c.* 157,464	*d.* 438,976
e. 32,768	*f.* 941,192	*g.* 912,673	*h.* 274,625
i. 29,791	*j.* 110,592	*k.* 753,571	*l.* 614,125
m. 42,875	*n.* 117,649	*o.* 405,224	*p.* 185,193

NOTE: When there are more than two figures in the root, let t of the formula represent all of the root known, and begin again with $3t^2u$, and repeat.

1. Find the cube root of 34,328,125.

$$\begin{array}{r} 3\quad 2\quad 5 \\ 34\cdot 328\cdot 125 \\ 27 = t^3 \\ 3t^2 = 27\overline{)73} \\ 54 = 3t^2u \\ \hline 192 \\ 36 = 3tu^2 \\ \hline 1568 \\ 8 = u^3 \\ 3072 = (3\times 32^2)\overline{)15601} \\ 15360 = 3t^2u \\ \hline 2412 \\ 2400 = 3tu^2 \\ \hline 125 \\ 125 = u^3 \\ \underline{\quad} \end{array}$$

2. Find the depth of a cubical box whose volume is 175,616 cubic inches.

3. A cubical block of stone contains 857,375 cubic inches. What is the area of one side?

4. A hall in the form of a cube contains 357,911 cubic feet. At \$.90 a square yard, how much will it cost to carpet the floor?

5. How long, wide, and high is a cubical pile of wood containing 32 cords?

6. A rectangular solid is 343 ft. long, 216 ft. wide, and 729 ft. high. Find the edge of a cube containing an equal number of cubical units.

7. A cube measures 5 in. on an edge. A second cube has 8 times the volume of the first. By how much does the length of an edge of the second cube exceed that of an edge of the first cube?

8. A cubical block of stone contains 50,653 cubic feet. What is its surface area?

9. What is the edge of a cube which contains as much as a solid 7 ft. long, 3 ft. 6 in. wide, and $1\frac{3}{4}$ ft. high?

10. What is the number of square inches in one face of a cubical block whose contents are 74,088 cubic inches?

11. Find the cube root of:

a. 43,614,208	*b.* 130,323,843	*c.* 354,894,912
d. 41,063,625	*e.* 303,464,448	*f.* 751,089,429
g. 14,348,907	*h.* 258,474,853	*i.* 27,081,081,027



1.	2.	3.	4.	5.	6.	7.	8.
+4	−4	+4	−4	+2	−2	+2	−2
+2	−2	−2	+2	+4	−4	−4	+4
+2	−2	+6	−6	−2	+2	+6	−6

The examples above may be illustrated as follows by using the diagram at the top:

(1). Put your pencil on the subtrahend, + 2, and move it to the minuend, + 4. In what direction did you move? Over how many spaces? The algebraic difference is + 2.

(3). Put your pencil on the subtrahend, − 2, and move it to the minuend, + 4. In what direction did you move? Over how many spaces? The algebraic difference is + 6.

(5). Put your pencil on the subtrahend, + 4, and move it to the minuend, + 2. In what direction did you move? Over how many spaces? The algebraic difference is − 2.

Illustrate 2, 4, 6, 7, and 8 in the same way.

From the illustrations we see that:

a. When you subtract a smaller number from a larger, the difference is always positive.

b. When you subtract a larger number from a smaller, the difference is always negative.

Learn: To subtract algebraic quantities, consider the sign of the subtrahend as changed from + to − or from − to +, and proceed as in addition.

Subtract:

9.	10.	11.	12.	13.
$7a$	$8x$	$-9y$	$-7b$	$-11z$
$4a$	$-5x$	$-4y$	$+2b$	$-7z$

14.	15.	16.	17.	18.
$14a$	$-2b$	$2c$	$2d$	$-11xy$
$-7a$	$8b$	$9c$	$-7d$	$-2xy$

1. Multiply a^3 by a^2.

$a^3 = a \times a \times a.$

$a^2 = a \times a.$

$\therefore a^3 \times a^2 = (a \times a \times a) \times (a \times a) = a \times a \times a \times a \times a = a^5.$

NOTE: The exponent of the product is found by adding the exponents of the factors.

Multiply:

2. x^7 by x^3
3. a^3c by ac^3
4. b^2x^2 by bx
5. a^2b by ab^2
6. x^4y by x^2y^2
7. xy^2z by x^2yz^3
8. $2a^2$ by $3a^2$
9. $6b$ by $4b^4$
10. $2ab$ by $3a^2b^2$
11. $4x$ by -3

$$\begin{array}{r} 4x \\ -3 \\ \hline -12x \end{array}$$

12. $-4a^2$ by $-7a$

$$\begin{array}{r} -4a^2 \\ -7a \\ \hline 28a^3 \end{array}$$

13. $-2x^2$ by $7x$

$$\begin{array}{r} -2x^2 \\ +7x \\ \hline -14x^3 \end{array}$$

Learn: In algebraic multiplication:

a. Like signs give a positive product.

b. Unlike signs give a negative product.

Multiply:

14. $2a^3c$ by $-3ac$
15. $-8xy$ by $2xy$
16. $-4xy^2$ by $-2xy$
17. $2ab$ by $3a^2b$
18. $2ac$ by $-2ac$
19. $6cd^2$ by $-2c^2d$
20. Multiply $7a - 4b - 6c$ by $2c$.

$$\begin{array}{r} 7a - 4b - 6c \\ 2c \\ \hline 14ac - 8bc - 12c^2 \end{array}$$

Multiply:

21. $x^2 + 2xy + y^2$ by $4xy$
22. $a^3 - 3a^2b + 4a^2b^2 + b^3$ by $2a^2b^2$
23. $x^4 - 4x^3y + 4x^2y^2 - 2xy + y^2$ by $3xy^2$
24. To $\frac{3}{5}$ of a certain number 8 was added, and the sum was 12 more than $\frac{1}{5}$ of the number. Find the number.
25. In a certain school of 840 pupils there are $\frac{3}{4}$ as many boys as girls. How many girls are there in the school?

AVOIRDUPOIS WEIGHT

16 ounces (oz.) = 1 pound (lb.)
100 pounds = 1 hundredweight (C.)
2000 pounds = 1 ton (T.)

A long ton is 2240 lb., used in buying coal at the mine and by custom-house officers in collecting duties. A barrel of flour weighs 196 lb. A bushel of oats weighs 32 lb., corn 56 lb., wheat or potatoes 60 lb.

TIME MEASURE

60 seconds (sec.) = 1 minute (min.)
60 minutes = 1 hour (hr.)
24 hours = 1 day (da.)
7 days = 1 week (wk.)
12 months = 1 year (yr.)

"Thirty days hath September,
April, June, and November."

The other months have 31 days, except February, which has 28 days in common years and 29 days in leap years. All years divisible by 4 or 400 are leap years. A decade is 10 years, and a century is 100 years.

LIQUID MEASURE

4 gills (gi.) = 1 pint (pt.)
2 pints = 1 quart (qt.)
4 quarts = 1 gallon (gal.)

A gallon contains 231 cu. in. For general purposes it is sufficiently accurate to call $7\frac{1}{2}$ gal. equal to a cubic foot. $31\frac{1}{2}$ gal. are considered a barrel, and 2 bbl. a hogshead.

DRY MEASURE

2 pints (pt.) = 1 quart (qt.)
8 quarts = 1 peck (pk.)
4 pecks = 1 bushel (bu.)

A bushel, even measure, contains 2150.42 cu. in. A bushel, heaped measure, contains 2688 cu. in. It is sufficiently accurate to say there are $1\frac{1}{4}$ cu. ft. in an even bushel, and $1\frac{1}{2}$ cu. ft. in a heaped bushel. The even bushel is used in measuring all kinds of grain, and the heaped bushel in measuring all kinds of fruits or vegetables.

TABLES FOR COUNTING

20 units = 1 score
12 units = 1 dozen (doz.)
12 dozen = 1 gross (gro.)

24 sheets of paper = 1 quire
20 quires = 1 ream

Paper is often sold by the pound instead of the quire or ream.

A folio is paper folded into 2 leaves in making books; a quarto or 4to is folded into 4 leaves; an octavo or 8vo into 8 leaves, and a duodecimo or 12mo into 12 leaves.

MEASURE OF LENGTH OR LONG MEASURE

12 inches (in.)	**= 1 foot (ft.)**
3 feet	**= 1 yard (yd.)**
$5\frac{1}{2}$ yards or $16\frac{1}{2}$ feet	**= 1 rod (rd.)**
320 rods or 5280 feet	**= 1 mile (mi.)**

NOTE: 6′ is often written for 6 ft., and 6″ for 6 in.

A *hand* is 4 in., used in measuring the height of horses.

A *fathom* is 6 ft., used in measuring the depth of water.

A *knot* or nautical mile is 1.15 miles, and a *league* is 3 miles, both used in measuring distances at sea.

A *pace* is 3 ft., used in estimating distances.

A *chain* is 66 ft., used by surveyors.

MEASURE OF SURFACE OR SQUARE MEASURE

144 square inches (sq. in.)	**= 1 square foot (sq. ft.)**
9 square feet	**= 1 square yard (sq. yd.)**
$30\frac{1}{4}$ square yards or $272\frac{1}{4}$ square feet	**= 1 square rod (sq. rd.)**
160 square rods	**= 1 acre (A.)**
640 acres	**= 1 square mile (sq. mi.)**

A *section* of land is 1 mile square.

A *square* is 100 sq. ft. of roofing.

MEASURE OF VOLUME OR CUBIC MEASURE

1728 cubic inches (cu. in.)	**= 1 cubic foot (cu. ft.)**
27 cubic feet	**= 1 cubic yard (cu. yd.)**
128 cubic feet	**= 1 cord (cd.)**

A *perch* is usually $24\frac{3}{4}$ cu. ft., used in measuring stones and masonry.

A *load* is 1 cu. yd., used by contractors in excavating earth for cellars, etc.

A *brick* is 8 in. long, 4 in. wide, and 2 in. thick; 22 bricks are estimated as equal to 1 cu. ft.

TABLE OF U. S. MONEY

10 mills = 1 cent (¢)
10 cents = 1 dime
10 dimes = 1 dollar ($)
10 dollars = 1 eagle (E.)

The standard of value in the U. S. is the dollar. The mill is not coined, but is expressed as a tenth of a cent. The coins of the U. S. are made of bronze, nickel, silver, and gold.

TABLE OF ENGLISH MONEY

4 farthings (far). = 1 penny (d.)
12 pence = 1 shilling (s.)
20 shillings = 1 pound (£)

The standard of value in England is the pound or sovereign. The farthing is not coined, but is expressed as a fraction of a penny.

The coins of England are made of copper, silver, and gold. The names and values of coins in Canada are the same as in the United States.

FRENCH MONEY

10 millimes = 1 centime
10 centimes = 1 decime
10 decimes = 1 franc

GERMAN MONEY

100 pfennigs = 1 mark
3 marks = 1 Thaler

TABLE OF ANGLES AND CIRCLES

60 seconds (″) = 1 minute (′)
60 minutes = 1 degree (°)
360 degrees = 1 circumference

Any part of the circumference of a circle is an **arc**. One quarter of the circumference is a **quadrant**.

SURVEYORS' MEASURE

7.92 inches = 1 link
100 links = 1 chain
80 chains = 1 mile

APOTHECARIES' WEIGHT

20 grains (gr.) = 1 scruple (sc. or ℈)
3 scruples = 1 dram (dr. or ʒ)
8 drams = 1 ounce (oz. or ℥)
12 ounces = 1 pound (lb. or ℔)
60 drops (gtt.) = 1 fluid dram (fʒ)
8 fluid drams = 1 fluid ounce (f℥)
16 fluid ounces = 1 pint (O.)
8 pints = 1 gallon (cong.)

